BRIN MATHEMATICS RESEARCH CENTER

Workshop:
Stochastic PDEs & Related Topics

NOVEMBER 14 - 16, 2022

Speakers

Jacob Bedrossian, University of Maryland
Le Chen, Auburn University
Ivan Corwin, Columbia University
Alexander Dunlap, NYU
Promit Ghosal, MIT
Yier Lin, University of Chicago
Sergey Lototsky, USC
Konstantin Matetski, Michigan State University
Jonathan Mattingly, Duke University
Carl Mueller, University of Rochester
Andrea Nahmod, University of Massachusetts Amherst
Hao Shen, University of Wisconsin Madison
Samy Tindel, Purdue University
Kevin Yang, UC Berkeley

About the Workshop

The goal of this workshop is to bring junior and promising scholars together with senior and established investigators, all working in the field of SPDEs. The workshop will provide the participants with opportunities to showcase their results and to prepare the grounds for future developments.

BRINMRC.UMD.EDU

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College Park, MD 20742

Organizers

Sandra Cerrai, University of Maryland
Yu Gu, University of Maryland
Shalin Parekh, University of Maryland

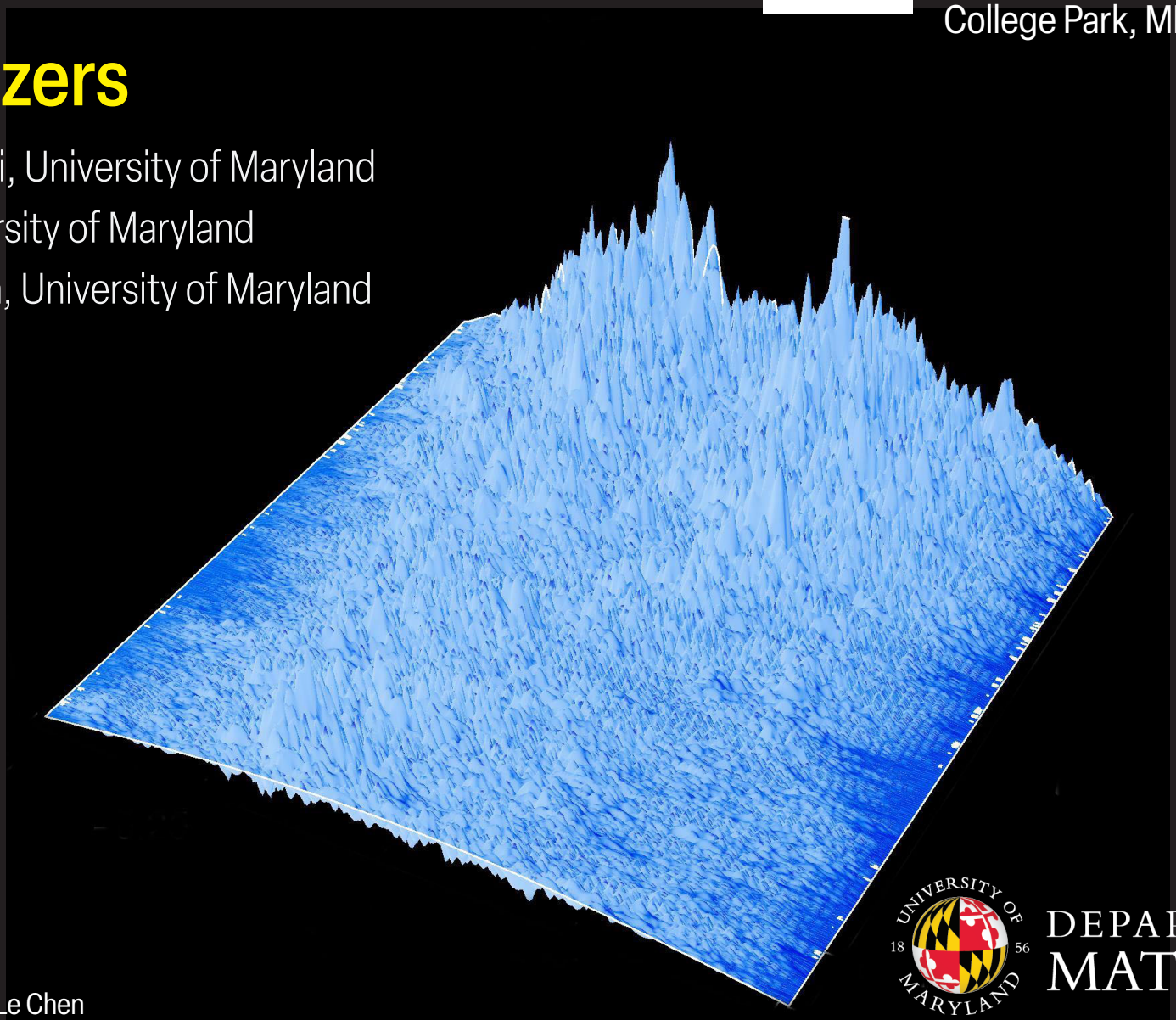

Image courtesy of Le Chen

DEPARTMENT OF MATHEMATICS

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Workshop Overview

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Organizing committee

Sandra Cerrai, University of Maryland

Yu Gu, University of Maryland

Shalin Parekh University of Maryland

Schedule at a Glance

	Monday	Tuesday	Wednesday
8:00			
	Breakfast	Breakfast	Breakfast
9:00	Carl Mueller	Andrea R Nahmod	Jonathan Mattingly
10:00	Sergey Lototsky	Konstantin Matetski	Jacob Bedrossian
11:00	Coffee break	Coffee break	Coffee break
	Alexander Dunlap	Promit Ghosal	Le Chen
12:00	Lunch	Lunch	Lunch
13:00			
14:00	Samy Tindel	Ivan Corwin	
15:00	Coffee break	Coffee break	
	Yier Lin	Kevin Yang	
16:00			
	Hao Shen		
17:00			

Workshop Schedule

Monday, November 14, 2022

8:30 - 8:55	Breakfast
8:55 - 9:00	Doron Levy (University of Maryland/Director, Brin MRC) *Opening*
9:00 - 9:45	Carl Mueller (University of Rochester) *Self-Avoiding Models of Moving Polymers and Surfaces*
10:00 - 10:45	Sergey Lototsky (USC) *Gaussian Free Fields and Stochastic Heat Equations*
10:45 - 11:15	Coffee break
11:15 - 12:00	Alexander Dunlap (NYU) *The nonlinear stochastic heat equation in the critical dimension*
12:00 - 2:00	Lunch
2:00 - 2:45	Samy Tindel (Purdue University) *Hyperbolic Anderson model in the Skorohod and rough settings*
2:45 - 3:15	Coffee break
3:15 - 4:00	Yier Lin (University of Chicago) *Some recent progress in the weak noise theory of the KPZ equation*
4:15 - 5:00	Hao Shen (University of Wisconsin Madison) *Stochastic Yang-Mills process in 2D and 3D*

Tuesday, November 15, 2022

8:30 - 9:00 Breakfast

9:00 - 9:45 Andrea R Nahmod (University of Massachusetts Amherst)
Invariant Gibbs measures for 2D NLS and 3D cubic NLW

10:00 - 10:45 Konstantin Matetski (Michigan State University)
Directed mean curvature flow in noisy environment

10:45 - 11:15 Coffee break

11:15 - 12:00 Promit Ghosal (MIT)
Fractal geometry of the parabolic Anderson model in two and three dimensions with white noise potential

12:00 - 2:00 Lunch

2:00 - 2:45 Ivan Corwin (Columbia University)
Boundaries for KPZ

2:45 - 3:15 Coffee break

3:15 - 4:00 Kevin Yang (UC Berkeley)
Time-dependent KPZ equation from non-equilibrium Ginzburg-Landau SDEs

Wednesday, November 16, 2022

8:30 - 9:00 Breakfast

9:00 - 9:45 Jonathan Mattingly (Duke University)
A randomly split model for Euler Dynamics

10:00 - 10:45 Jacob Bedrossian (University of Maryland)
Strictly positive lower bounds on the top Lyapunov exponent of Galerkin-Navier-Stokes, Lorenz-96, and other stochastic differential equations

10:45 - 11:15 Coffee break

11:15 - 12:00 Le Chen (Auburn University)
Invariant measures for nonlinear stochastic heat equation with no drift term

12:00 - 1:00 Lunch

1:00 - 1:05 Workshop Closing

Abstracts of talks

Self-Avoiding Models of Moving Polymers and Surfaces

Carl Mueller

University of Rochester

Monday, November 14, 2022 @ 9:00 AM

This is joint work with Eyal Neuman.
Polymer models give rise to some of the most challenging problems in probability and statistical physics. We typically model a polymer using a random walk, where the time parameter n of the walk represents distance along the polymer starting from one end. That is, we imagine that the polymer is built up by adding molecules one by one at random angles. We usually include a self-avoidance term, reflecting the idea that different parts of the polymer cannot be in the same place at the same time. A difficult problem, unsolved in the most important physical cases, is to predict the end-to-end distance or radius of the polymer.
In this talk, I will discuss two extensions of the random polymer model.
1. Moving polymers can be modeled by stochastic partial differential equa- tions. If the polymer takes values in one-dimensional Euclidean space, we give fairly sharp upper and lower bounds for its radius. We find that there is more stretching than in typical one-dimensional polymer models that do not have time dependence.
2. Random surfaces can be modeled by elastic manifolds, also called dis- crete Gaussian free fields. Free fields originate in quantum field theory. If the dimensions of the parameter space and the range are the same, we can derive bounds on the radius of the polymer. These bounds are fairly sharp in two dimensions. We will explain the models mentioned above and give an outline of our proof techniques.

Gaussian Free Fields and Stochastic Heat Equations

Sergey Lototsky

USC

Monday, November 14, 2022 @ 10:00 AM

The Gaussian free field in a bounded planar region appears as a scaling limit of certain discrete models, but it is also a stationary solution of the heat equation with additive space-time white noise. This observation naturally leads to other types of Gaussian free fields, in both bounded and unbounded domains, and to interesting phenomena in dimensions one and two.

The nonlinear stochastic heat equation in the critical dimension

Alexander Dunlap

NYU

Monday, November 14, 2022 @ 11:15 AM

I will discuss a two-dimensional stochastic heat equation with a nonlinear noise strength, and consider a limit in which the correlation length of the noise is taken to 0 but the noise is attenuated by a logarithmic factor. I will discuss how pointwise statistics of this equation can be related to a forward-backward stochastic differential equation (FBSDE) depending on the nonlinearity, as long as the nonlinearity is such that the FBSDE can be solved for long enough with Lipschitz decoupling function. I will also discuss conditions on the nonlinearity for this property to hold, and explain several cases in which the FBSDE can be solved explicitly. No previous knowledge of forward-backward SDEs will be assumed. This talk will be based on current joint work with Cole Graham and older joint work with Yu Gu.

Hyperbolic Anderson model in the Skorohod and rough settings

Samy Tindel

Purdue University

Monday, November 14, 2022 @ 2:00 PM

In this talk I will start by giving a brief overview of some standard results concerning the stochastic heat equation, for which existence and uniqueness results are well established for a large class of Gaussian noises. Then I will describe some recent advances aiming at a proper definition of noisy wave equations, when specialized to a bilinear setting (called hyperbolic Anderson model). First I will focus on the so-called Skorohod setting, where an explicit chaos decomposition of the solution is available. A good control of the chaos expansion is then achieved thanks to an exponentiation trick. Next I will turn to a pathwise approach, which is based on a novel Strichartz type estimate for the wave operator. If possible I will show the main steps of this analytic estimate.

Some recent progress in the weak noise theory of the KPZ equation

Yier Lin

University of Chicago

Monday, November 14, 2022 @ 3:15 PM

In this talk, we will study the Freidlin–Wentzell LDP for the KPZ equation using the variational principle. Such an approach goes under the name of the weak noise theory in physics. We will explain how to extract various limits of the most probable shape of the KPZ equation in the setting of the Freidlin–Wentzell LDP. The talk is based on joint work with Pierre Yves Gaudreau Lamarre and Li-Cheng Tsai.

Stochastic Yang-Mills process in 2D and 3D

Hao Shen

University of Wisconsin Madison

Monday, November 14, 2022 @ 4:15 PM

We will discuss stochastic quantization of the Yang-Mills model in 2 and 3 space dimensions. This includes constructing the Langevin dynamic for the formal Yang-Mills measure, defining the state space of gauge orbits, proving gauge equivariance of the dynamic, and making sense of Wilson loop observables in this context. We will also discuss some future directions. The talk is based on several works mostly joint with A.Chandra, I.Chevyrev, and M.Hairer.

Invariant Gibbs measures for 2D NLS and 3D cubic NLW

Andrea R Nahmod

University of Massachusetts Amherst

Tuesday, November 15, 2022 @ 9:00 AM

In this talk we discuss recent developments in the study of propagation of randomness under the flow of dispersive PDE. In particular we present a non-technical overview of recent works that led to the resolution of two open problems concerning Gibbs measure invariance for the 2D NLS with arbitrary wave interactions (joint with Yu Deng and Haitian Yue) and for the 3D cubic NLW (joint with Bjoern Bringmann, Yu Deng and Haitian Yue). The first one is proved using the method of random averaging operators, while the second one relies on the theory of random tensors in conjunction with other techniques, such as paracontrolled calculus and heat-wave analysis.

Directed mean curvature flow in noisy environment

Konstantin Matetski

Michigan State University

Tuesday, November 15, 2022 @ 10:00 AM

We consider the directed mean curvature flow on the plane in a weak Gaussian random environment. We prove that, when started from a sufficiently flat initial condition, a rescaled and recentred solution converges to the Cole-Hopf solution of the KPZ equation. This result follows from the analysis of a more general system of nonlinear SPDEs driven by inhomogeneous noises, using the theory of regularity structures. This is joint work with Martin Hairer and Andris Gerasimovics.

Fractal geometry of the parabolic Anderson model in two and three dimensions with white noise potential

Promit Ghosal

MIT

Tuesday, November 15, 2022 @ 11:15 AM

Parabolic Anderson model is one of the prototypical models of random walk in random media which bears lot of applications to real-world problems in sciences, like astrophysics, magnetohydrodynamics, chemical reactions etc. In this talk, we discuss about the fractal geometry of the peaks of the parabolic Anderson model (PAM) with Gaussian (spatial) white noise in two and three dimension. We show that the spatial and spatial-temporal peaks of the PAM are macroscopically multifractal. More precisely, we compute the macroscopic Hausdorff dimension of those peaks. As a by-product, we obtain the spatial asymptotics of the PAM. This talk will be based on a joint work with Jaeyun Yi.

Boundaries for KPZ

Ivan Corwin

Columbia University

Tuesday, November 15, 2022 @ 2:00 PM

I will describe the effect of boundary conditions on the behavior of KPZ class models, mainly in terms of their invariant measures which remarkably relate to versions of 1d Liouville quantum gravity. This is based on joint works with Hao Shen, Alisa Knizel and Guillaume Barraquand.

Time-dependent KPZ equation from non-equilibrium Ginzburg-Landau SDEs

Kevin Yang

UC Berkeley

Tuesday, November 15, 2022 @ 3:15 PM

This talk has two goals. The first is the derivation of a time-dependent KPZ equation (TDKPZ) from a time-inhomogeneous Ginzburg-Landau model, which describes, among many other things, stochastic dynamics of ferromagnets whose physics have been observed to be time-dependent (due to Eddy currents). The TDKPZ has a nonlinear twist that is not seen in the usual KPZ equation, making it a more interesting SPDE.
The second goal is universality of the method for deriving (TD)KPZ, which should work beyond Ginzburg-Landau. In particular, we answer a question of deriving (TD)KPZ from (classical) asymmetric particle systems under natural fluctuation-versions of the assumptions in Yau's relative entropy method; the only additional assumption we need is a log-Sobolev inequality. Time permitting, future directions (of pure and applied flavors) will be discussed.

A randomly split model for Euler Dynamics

Jonathan Mattingly

Duke University

Wednesday, November 16, 2022 @ 9:00 AM

I will discuss a random model for Euler dynamics based on a random splitting of the vector fields. I will discuss what can be said about the system's ergodic properties including its Lyapunov exponents and how it behaves under various driving forces with balancing dissipation.

Strictly positive lower bounds on the top Lyapunov exponent of Galerkin-Navier-Stokes, Lorenz-96, and other stochastic differential equations

Jacob Bedrossian

University of Maryland

Wednesday, November 16, 2022 @ 10:00 AM

We review our recent joint work with Alex Blumenthal and Sam Punshon-Smith, which introduced methods for obtaining strictly positive lower bounds on the top Lyapunov exponent of high-dimensional, stochastic differential equations such as the weakly damped Lorenz-96 (L96) model or Galerkin truncations of the 2d Navier-Stokes equations. This hallmark of chaos has long been observed in these models, however, no mathematical proof had previously been made for either deterministic or stochastic forcing. The method we proposed combines (A) a new identity connecting the Lyapunov exponents to a Fisher information of the stationary measure of the Markov process tracking tangent directions (the so-called "projective process"); and (B) an L1 -based hypoelliptic regularity estimate to show that this (degenerate) Fisher information is an upper bound on some fractional regularity. We will briefly review the concept of Lyapunov exponents in SDEs in general and this method in particular.

Invariant measures for nonlinear stochastic heat equation with no drift term

Le Chen

Auburn University

Wednesday, November 16, 2022 @ 11:15 AM

In this talk, we will present a recent joint work with Dr. Nicholas Eisenberg (arXiv:2209.04771). This paper deals with the long term behavior of the solution to the nonlinear stochastic heat equation $\partial u/\partial t - \frac{1}{2}\Delta u = b(u)\dot{W}$, where b is assumed to be a globally Lipschitz continuous function and the noise $\dot{W}$ is a centered and spatially homogeneous Gaussian noise that is white in time. Using the moment formulas obtained in Chen & Kim [10] and Chen & Huang [9], we identify a set of conditions on the initial data, the correlation measure and the weight function ρ, which will together guarantee the existence of an invariant measure in the weighted space $L^2_\rho(\mathbb{R}^d)$. In particular, our result includes the *parabolic Anderson model* (i.e., the case when $b(u) = \lambda u$) starting from the Dirac delta measure.

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

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Le Chen, Auburn University
Leonid Koralov, University of Maryland
Mengzi Xie, University of Maryland
Panqiu Xia, Auburn University
Promit Ghosal, MIT
Ran Tao, University of Maryland
Samy Tindel, Purdue University
Sandra Cerrai, University of Maryland
Sergey Lototsky, USC
Shalin Parekh, University of Maryland
Shuo Yan, University of Maryland
Victor Gardner, Duke University
Wen-Tai Hsu, University of Maryland
Yichun Zhu, University of Maryland
Yier Lin, University of Chicago
Yu Gu, University of Maryland