BRIN MATHEMATICS RESEARCH CENTER

DEPARTMENT OF MATHEMATICS

Low Complexity Dynamical Systems

October 2-6, 2023

About the Workshop

The goal of the workshop is to bring together experts in various different areas of low complexity systems research to survey the recent progress, aiming to enhance cross-subfield collaboration and connect the various research programs, ideally leading to potential progress on several long-standing conjectures.

Organizers

Darren Creutz, US Naval Academy
Adam Kanigowski, University of Maryland
Rodrigo Trevino, University of Maryland

Speakers

Terry Adams, IEEE
Melodie Andrieu, Bar-Ilan University
Sebastian Barbieri, Universidad de Santiago de Chile
Julien Cassaigne, Aix-Marseille Université
Daniel Coronel, Pontificia Universidad Católica de Chile
Van Cyr, Bucknell University
Sebastian Donoso, Universidad de Chile
Bastián Espinoza, Universidad de Chile
Sebastien Ferenczi, Aix-Marseille Université
Jonathan Fickenscher, Princeton University
Felipe Garcia Ramos, Universidad Autónoma de San Luis Potosí
Jennifer Jones, Northwestern University
Bryna Kra, Northwestern University
Katy Loyd, Northwestern University
Alejandro Maass, Universidad de Chile
Ronnie Pavlov, University of Denver
Samuel Petite, Université de Picardie Jules Verne
Scott Schmieding, Penn State University
Kitty Yang, University of North Carolina Asheville

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Schedule at a Glance

	Monday	Tuesday	Wednesday	Thursday	Friday
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11:00	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
	Scott Schmieding	Ronnie Pavlov	Julien Cassaigne	Sebastián Donoso	Samuel Petite
12:00	Lunch	Lunch (on your own)	Katy Loyd	Lunch	Mélodie Andrieu
13:00			Lunch and sightseeing (on your own)		Lunch
14:00	Sébastien Ferenczi	Felipe García-Ramos		Bryna Kra	
15:00	Coffee Break	Nelson Moll		Coffee Break	
	Kitty Yang			Terry Adams	
16:00	Jennifer Jones	Bastián Espinoza		Noah Kravitz	
17:00	High Tea				
18:00					

Workshop Overview

Multiple notions of low complexity exist for dynamical systems: low structural complexity, e.g. symbol substitution systems and interval exchange maps; low dynamical complexity, e.g. (partial) rigidity, countable (or finite) ergodic measures, loosely Bernoulli property, lack of weak mixing and discrete spectrum; and low word complexity subshifts.

Recent work, by various authors, established that (sub)linear word complexity implies low complexity in a variety of other senses, e.g. being substitutive of finite alphabet rank, having at most countably many ergodic measures, necessarily being partially rigid, and so on.

Two major conjectures relate forms of low complexity: the S-adic conjecture, which asserts that there is an explicit relationship between (sub)linear word complexity and a substitutive structure, and the Pisot conjecture, which asserts that, in the context of substitution systems, discrete spectrum is equivalent, roughly, to a specific form of algebraic substitutive structure (and presumably these are also implied, in some sense, by a word complexity property).

The goal of the workshop is to bring together experts in various different areas of low complexity systems research to survey the recent progress, aiming to enhance cross-subfield collaboration and connect the various research programs, ideally leading to potential progress on the aforementioned and other longstanding conjectures.

Organizing committee

Darren Creutz, US Naval Academy

Adam Kanigowski, University of Maryland

Rodrigo Treviño, University of Maryland

Workshop Schedule

Monday, October 2, 2023

9:00 - 9:25 Breakfast

9:25 - 9:30 Sandra Cerrai (University of Maryland)
Opening

9:30 - 10:20 Alejandro Maass (Universidad de Chile)
An overview of some relevant aspects and questions related to finte rank S-adic systems (and/or finite topological rank Cantor minimal systems)

10:30 - 11:00 Coffee Break

11:00 - 11:50 Scott Schmieding (Penn State University)
Automorphism groups of some low complexity subshifts

12:00 - 1:30 Lunch

1:30 - 2:20 Sébastien Ferenczi (Aix-Marseille Université)
Languages of general interval exchanges

2:30 - 3:00 Coffee Break

3:00 - 3:50 Kitty Yang (University of North Carolina)
Mapping class group of low complexity subshifts

4:00 - 4:30 Jennifer Jones (Northwestern University)
The stabilized automorphism group of low complexity subshifts

4:30 - 5:00 High Tea

Tuesday, October 3, 2023

9:00 - 9:30 Breakfast

9:30 - 10:20 Van Cyr (Bucknell University)
Strong approximations of shifts and ergodic properties of their measures

10:30 - 11:00 Coffee Break

11:00 - 11:50 Ronnie Pavlov (University of Denver)
Subshifts of very low word complexity

12:00 - 2:00 Lunch (on your own)

2:00 - 2:50 Felipe García-Ramos (Universidad Autunoma de San Luis Potosl / Jagiellonian Universi
Cubic independence and almost finite to one extensions

3:00 - 3:30 Nelson Moll (University of Maryland)
Speed of weak mixing for the Chacon Map

3:40 - 4:10 Bastián Espinoza (Universidad de Chile)
A structure theorem for low complexity subshifts

Wednesday, October 4, 2023

9:00 - 9:30 Breakfast

9:30 - 10:20 Jonathan Fickenscher (Princeton University)
Generic measures for recurrent subshifts with strictly linear complexity

10:30 - 11:00 Coffee Break

11:00 - 11:50 Julien Cassaigne (CNRS, Aix-Marseille Université)
The Heinis spectrum

12:00 - 12:30 Katy Loyd (Northwestern University)
New points in the Heinis spectrum

12:30 - 12:35 Group Photo

12:35 - 7:00 Lunch and sightseeing (on your own)

7:00 - 9:00 Conference Dinner

Thursday, October 5, 2023

9:00 - 9:30	Breakfast
9:30 - 10:20	Sebastián Barbieri (Universidad de Santiago de Chile) *Sturmian configurations through asymptotic pairs*
10:30 - 11:00	Coffee Break
11:00 - 11:50	Sebastián Donoso (Universidad de Chile) *A geometric approach to nonexpansive directions*
12:00 - 1:30	Lunch
1:30 - 2:20	Bryna Kra (Northwestern University) *Complexity of combinatorial configurations*
2:30 - 3:00	Coffee Break
3:00 - 3:50	Terry Adams (IEEE) *Typical values for slow entropy*
4:00 - 4:30	Noah Kravitz (Princeton University) *Nivat's Conjecture and low-complexity grid colorings*

Friday, October 6, 2023

9:00 - 9:30 Breakfast

9:30 - 10:20 Daniel Coronel (Pontificia Universidad Catulica de Chile)
Thermodynamics Formalism for the Manneville-Pomeau Map with Holder Potentials

10:30 - 11:00 Coffee Break

11:00 - 11:50 Samuel Petite (Universite de Picardie Jules Verne)
Normalizer of odometers and automatic $\mathbb{Z}^d$-arrays

12:00 - 12:50 Mélodie Andrieu (Bar-Ilan University)
Complexities of words generated by a billiard in the hypercube

12:50 - 1:00 Workshop Closing

1:00 - 2:00 Lunch

Abstracts of talks

An overview of some relevant aspects and questions related to finte rank S-adic systems (and/or finite topological rank Cantor minimal systems)

Alejandro Maass

Universidad de Chile

Monday, October 2, 2023 @ 9:30 AM

The study of minimal Cantor systems became very relevant with the work in the 1990s by Giordano, Putnam and Skau on the characterization of orbital equivalence using algebraic properties of Bratteli-Vershik diagrams. Relevant classes are substitutive, linearly recurrent or finite topological rank systems. Since the 1990s these structures have been key to characterize classes of systems and their factors (Hedlund-Morse-like structural properties), to describe dynamical aspects such as the spectrum of such systems (continuous and measurable eigenvalues), the group of automorphisms or to understand rigidity properties, among many. On the other hand, from Downarowicz-Maass result one can establish that in the case of finite topological rank that Cantor minimal systems are subshifts or odometers, which allows to transit into the world of S-adic systems and to transfer many ideas from the world of Bratteli-Vershik diagrams. In this talk we will focus on establishing this relation, in particular for low complexity (non-superlinear) systems, and on describing recently established dynamical properties: eigenvalues of finite rank minimal Cantor systems and group of automorphisms.

Automorphism groups of some low complexity subshifts

Scott Schmieding

Penn State University

Monday, October 2, 2023 @ 11:00 AM

I'll discuss some joint work with Pavlov where we characterize the class of countable groups which can arise as the automorphism group of a subshift whose complexity grows like $n(\log\log n)^o(1)$ along a subsequence.

Languages of general interval exchanges

Sébastien Ferenczi

Aix-Marseille Université

Monday, October 2, 2023 @ 1:30 PM

The languages generated by interval exchange transformations have been characterized by Ferenczi-Zamboni (2008) and Belov-Cernyatev (2010) under some extra conditions on the system. Lifting these conditions leads us to consider successively natural codings of standard interval exchange transformations, natural codings of affine interval exchange transformations, grouped codings of affine interval exchange transformations, and natural codings of generalized interval exchange transformations. We show that these four classes of languages are strictly increasing, and give necessary and/or sufficient (but not all equally explicit) combinatorial criteria to describe each of them. The main criterion to characterize these languages, called the order condition, is also linked to the problem of clustering words for the Burrows-Wheeler transform used in data compression.

Mapping class group of low complexity subshifts

Kitty Yang

University of North Carolina

Monday, October 2, 2023 @ 3:00 PM

Given a dynamical system (X, f), the automorphism group can be viewed as the symmetry group of X with respect to conjugacy. By considering the flow equivalence relation, we can similarly construct the mapping class group, the symmetry group of X with respect to flow equivalence, up to isotopy. We show that the mapping class group of minimal, linear complexity subshifts satisfying a technical condition (no infinitesimals) are virtually abelian. In the case of primitive substitutive systems, we show that the mapping class group is virtually $\mathbb{Z}$.
This is joint work with Scott Schmieding.

The stabilized automorphism group of low complexity subshifts

Jennifer Jones

Northwestern University

Monday, October 2, 2023 @ 4:00 PM

The stabilized automorphism group of a subshift (X, T) is the group of all self-homeomorphisms of X that commute with some power of T. For transitive subshifts of linear complexity with an additional restriction, we characterize the stabilized automorphism group and show that it fully determines the rational eigenvalues of (X, T). Time permitting, we will talk about some other low complexity cases.

Strong approximations of shifts and ergodic properties of their measures

Van Cyr

Bucknell University

Tuesday, October 3, 2023 @ 9:30 AM

The Krylov-Bogolyubov theorem guarantees that every topological Z-system has a measure invariant under the dynamics. But what if we seek a measure with additional properties? In this talk I will survey some recent research joint with Kra-Petite that investigates this question and specifically when a subshift has a measure invariant under not only the shift but all automorphisms of the system. A key tool will be the rate of approximation of a shift by the terms of its natural cover by subshifts of finite type, and when low complexity systems have such an approximation.

Subshifts of very low word complexity

Ronnie Pavlov

University of Denver

Tuesday, October 3, 2023 @ 11:00 AM

The word complexity function $p(n)$ of a subshift X measures the number of n-letter words appearing in sequences in X, and X is said to have linear complexity if $p(n)/n$ is bounded. It's been known since work of Ferenczi that linear complexity highly constrains the dynamical behavior of a subshift. In recent work with Darren Creutz, we show that if X is a transitive subshift with $\limsup p(n)/n < 3/2$, then X is measure-theoretically isomorphic to a compact abelian group rotation. Our proof relies on a substitutive/S-adic decomposition for such subshifts.
I'll discuss this result, as well as several ways in which 3/2 turns out to be a key threshold (for $\limsup p(n)/n$) for several different types of dynamical behavior.

Cubic independence and almost finite to one extensions

Felipe García-Ramos

Universidad Autunoma de San Luis Potosl / Jagiellonian University

Tuesday, October 3, 2023 @ 2:00 PM

We will study combinatorial dynamical conditions that imply that a system is an almost finite to one extension of a nilsystem.

Speed of weak mixing for the Chacon Map

Nelson Moll

University of Maryland

Tuesday, October 3, 2023 @ 3:00 PM

We first consider a non-primitive substitution subshift that is conjugate to the Chacon map. We then derive spectral estimates for a particular subshift and the speed of weak mixing for a class of observables with certain regularity conditions. After, we use these results to find the speed of weak mixing for the Chacon map on the interval and show that this bound is essentially sharp. The talk is based on my arXiv paper: https://arxiv.org/abs/2308.00823

A structure theorem for low complexity subshifts

Bastián Espinoza

Universidad de Chile

Tuesday, October 3, 2023 @ 3:40 PM

An idea that became unavoidable to study zero entropy symbolic dynamics is that the dynamical properties of a system induce in it a combinatorial structure. An old problem addressing this intuition is finding a structure theorem for linear-growth complexity subshifts using the S-adic formalism. It is known as the S-adic conjecture and motivated several influential results in the theory. In this presentation, I will present an S-adic structure for this class and show how this provides a unified framework and simplified proofs of several known results.

Generic measures for recurrent subshifts with strictly linear complexity

Jonathan Fickenscher

Princeton University

Wednesday, October 4, 2023 @ 9:30 AM

In this talk, we will discuss the relationship between the complexity function of a subshift and the generic measures of the system. Generic measures are those that admit an orbit whose time averages equal the space averages for each integrable function. So ergodic measures are generic by the pointwise ergodic theorem, but not every generic measure is ergodic. Previously, V. Cyr and B. Kra gave a bound on the number of generic measures based on upper and lower linear growth rates of the complexity function, and in prior joint work with M. Damron the speaker explored bounds on the number of ergodic measures. The speaker's current work considers strict linearity, starting with the assumption that the linear growth rate converges and then imposing even stronger conditions, to determine improvements to these bounds in the generic measure setting. We explore the sharpness of these bounds by constructing many classes of examples using Rauzy Graphs, which are defined by the language of the subshift.

The Heinis spectrum

Julien Cassaigne

CNRS, Aix-Marseille Université

Wednesday, October 4, 2023 @ 11:00 AM

Many families of infinite words (or of subshifts) have a subword complexity function $p(n)$ that grows linearly. It has sometimes a very simple form (such as $n+1$, $2n+1$, etc.), but often a more complicated behaviour, as for the Thue-Morse word. In his Ph.D. thesis, Alex Heinis introduced the set Ω of pairs (α, β) such that $\alpha = \liminf p(n)/n$ and $\beta = \limsup p(n)/n$ for some infinite word. For instance, the Thue-Morse word gives the point $(3, 10/3)$. But not every point with $\alpha \leq \beta$ can be obtained, and it is a challenge to characterize points in Ω. We present some properties of this set, and some questions that we find interesting.

New points in the Heinis spectrum

Katy Loyd

Northwestern University

Wednesday, October 4, 2023 @ 12:00 PM

It has been long studied which functions can arise as the complexity function, $p(n)$, of an infinite sequence. In 2001, Heinis studied allowable limiting behaviors, producing conditions on the pairs of real numbers that can arise as the liminf and limsup of the sequence $p(n)/n$. The collection of all allowable such pairs is called the Heinis spectrum, and many open questions about this set still remain. In this talk, we will discuss how the constructive nature of S-adic sequences can be leveraged to compute their complexity and utilize this to give an explicit S-adic construction producing new points in the Heinis spectrum.

Sturmian configurations through asymptotic pairs

Sebastián Barbieri

Universidad de Santiago de Chile

Thursday, October 5, 2023 @ 9:30 AM

We will present a series of results which characterize multidimensional Sturmian configurations through pairs of asymptotic configurations that satisfy a strong complexity assumption. As a corollary of our results we will derive a formula to compute the complexity of a multidimensional Sturmian configuration on any support which is connected through the canonical generators of Z^d. This is joint work with S. Labbe and S. Starosta.

A geometric approach to nonexpansive directions

Sebastián Donoso

Universidad de Chile

Thursday, October 5, 2023 @ 11:00 AM

In this talk, I will introduce a new framework for studying the notion of nonexpansivity in topological dynamics, which is valid for all f.g. group actions.
In particular, I will explain a new result that simultaneously generalizes Schwartzmanis and Boyle-Lindis theorems on nonexpansivity of subspaces. If time allows, I will show some applications in symbolic and Cantor dynamics and comment on open questions. This is based on a joint work with Alejandro Maass and Samuel Petite.

Complexity of combinatorial configurations

Bryna Kra

Northwestern University

Thursday, October 5, 2023 @ 1:30 PM

Resolving a conjecture of Erdos and Turan from the 1930's, in the 1970's Szemeredi showed that a set of integers with positive upper density contains arbitrarily long arithmetic progressions. Soon thereafter, Furstenberg used ergodic theory to gave a new proof of this result, leading to the development of combinatorial ergodic theory. These tools have led to uncovering many new patterns that occur in any sufficiently large set of integers and each of these patterns is controlled, in some precise sense, by a structure in the associated (topological or measurable) system. We give an overview of the method and discuss the complexity of these associated systems.

Typical values for slow entropy

Terry Adams

IEEE

Thursday, October 5, 2023 @ 3:00 PM

This talk will discuss the notion of slow entropy which was originally introduced by Katok and Thouvenot. Given a nontrivial sub-exponential rate function, we show the lower slow entropy is typically zero, and the upper slow entropy is typically infinite. Also, we show for any sub-exponential rate function, there exists a rigid ergodic measure preserving transformation T such that its lower slow entropy is infinite with respect to that rate function. A comparison is given to other entropy notions such as entropy convergence rate.

Nivat's Conjecture and low-complexity grid colorings

Noah Kravitz

Princeton University

Thursday, October 5, 2023 @ 4:00 PM

A classical result of Morse and Hedlund says that if a coloring of the integers has at most n distinct patterns of length n for some n, then the coloring must be periodic (with period at most n). The naïve higher-dimensional generalization of this statement (with patterns in intervals replaced by patterns in boxes) fails in three or more dimensions, and the two-dimensional problem is known as Nivat's Conjecture. I will survey recent work on this conjecture and explain how it relates to various notions of low complexity for grid colorings. I will also draw parallels with Bhattacharya's resolution of the Periodic Tiling Conjecture in two dimensions. Based on ongoing joint work with Rachel Greenfeld.

Thermodynamics Formalism for the Manneville-Pomeau Map with Holder Potentials

Daniel Coronel

Pontificia Universidad Catulica de Chile

Friday, October 6, 2023 @ 9:30 AM

In this presentation, I will be discussing the thermodynamics formalism applied to the Manneville-Pomeau map with Holder potentials. I will illustrate that for any Holder potential, the pressure exhibits, at most, one phase transition in temperature. Furthermore, I will provide a topological insight into the space of potentials, categorized according to equilibrium state properties.

Normalizer of odometers and automatic $\mathbb{Z}^d$-arrays

Samuel Petite

Universite de Picardie Jules Verne

Friday, October 6, 2023 @ 11:00 AM

For a $\mathbb{Z}^d$ topological dynamical system $(Z, T, \mathbb{Z}^d)$ an *isomomorphism*, is a self-homeomorphism $\phi : X \to X$ such that for some matrix $M \in GL(d, \mathbb{Z})$ and any $n \in \mathbb{Z}^d$, $\phi \circ T^n = T^{Mn} \circ \phi$, where T^n denote the self-homeomorphism of X given by the action of $n \in \mathbb{Z}^d$.
The collection of all the isomorphisms forms a group that is the normalizer of the set of transformations T^n.In the one-dimensional case isomorphisms corresponds to the notion of *flip conjugacy* of dynamical systems and by this fact are also called *reversing symmetries*.
These isomorphsims are not well understood even for classical systems. In this talk we will present a description of them for odometers and more precisely for $\mathbb{Z}^2$-constant base odometers, that is surprisingly not simple. We deduce a complete description of the isomorphsims of some $\mathbb{Z}^2$ minimal substitution subshifts. Thanks this, we will give the first example known of a minimal zero-entropy subshift with the largest possible normalizer group.
This is a joint work with Christopher Cabezas (Univ. de Liege).

Complexities of words generated by a billiard in the hypercube

Mélodie Andrieu

Bar-Ilan University

Friday, October 6, 2023 @ 12:00 PM

Sturmian words form a class of binary infinite words which sheds light, through its equivalent definitions, on remarkable interactions between *combinatorics*, *dynamical systems*, and *number theory*. It gives rise to several generalizations over the d-letter alphabet for $d \geq 3$, depending on the considered definition: Arnoux-Rauzy and episturmian words, other words associated with multidimensional continued fraction algorithms, interval exchange transformations, polygonal or cubic billiard words. A large program, initiated in the 80s, is to determine whether or not some of these classes of words/subshifts coincide, and *which characteristic properties of Sturmian words they still satisfy.*

This talk focuses on one of the dynamical representations of Sturmian words: as words generated by a billiard in a square table, which generalizes itself to a *billiard in the cube*, and in the cube of dimension d; and on two combinatorial quantities which characterize Sturmian words: the *subword complexity* and the *abelian complexity*.
The talk is motivated by the following question: are the subword and abelian complexities of cubic billiard words in dimension d *minimal* among the complexities of d-ary words, as is the case for Sturmian words, *i.e.,* when $d = 2$?

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

Mélodie Andrieu, Bar-Ilan University
Sebastián Barbieri, Universidad de Santiago de Chile
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Daniel Coronel, Pontificia Universidad Catulica de Chile
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