



Mathematical Models of Electronic Transport & Phases in Low-Dimensional Materials

March 11-15, 2024

ABOUT THE WORKSHOP

The goal of this interdisciplinary workshop is to bring together mathematicians, and theoretical and applied physicists with the purpose of identifying and discussing new mathematical problems of physical importance in electronic transport.

ORGANIZERS

Svetlana Jitomirskaya, University of California, Irvine
Mitchell Luskin, University of Minnesota
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Lingrui Ge, Peking University
Michael Hott, University of Minnesota
Ilya Kachkovskiy, Michigan State University
Efthimios Kaxiras, Harvard University
Tianyu Kong, University of Minnesota
Patrick Ledwith, Harvard University
Wencai Liu, Texas A&M University
Matthias Maier, Texas A&M University
Daniel Massatt, Louisiana State University
Matt Powel, Georgia Institute of Technology

Solomon Quinn, University of Minnesota
Angel Rubio, Flatiron Institute
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Tobias Stauber, ICMM, Spain
Kevin Stubbs, University of California, Berkeley
Grigory Tarnopolsky, Carnegie Mellon University
Oskar Vafek, Florida State University
Alexander Watson, University of Minnesota
Michael Weinstein, Columbia University
Mengxuan Yang, University of California, Berkeley
Qi Zhou, Nankai University
Xiaowen Zhu, University of Washington
Zoe Zhu, Stanford University



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Schedule at a Glance

	Monday	Tuesday	Wednesday	Thursday	Friday
8:00					
	Breakfast	Breakfast	Breakfast	Breakfast	Breakfast
9:00	Kaxiras	Stauber	Massatt	Ge	Liu
10:00	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
	Cances	Weinstein	Vafek	Zhou	Powell
11:00	Ledwith	Hott	Zhu	Maier	Zhu
12:00	Lunch at Brin	Lunch at Brin	Lunch at Brin	Lunch (on your own)	Kachkovskiy
13:00					Lunch (on your own)
14:00	Bernevig	Stubbs	Fu	Fayad (3206 Kirwan Hall)	
15:00	Coffee Break	Coffee Break	Coffee Break	Jitomirskaya (3206 Kirwan Hall)	
	Watson	Quinn	Tarnopolsky	High Tea (3206 Kirwan Hall)	
16:00		Kong	Bal		
17:00					
18:00					

All talks will be held in the Brin MRC center, located on the fourth floor of the CSIC building. Bassam Fayad and Svetlana Jitomirskaya talks will be in the Math Colloquium room, 3206 Kirwan Hall.

Workshop Overview

Low-dimensional moiré materials provide a rich testing bed for exploring novel quantum phenomena of electronic transport. For example, twisted bilayer graphene has attracted much attention because of the electronic phases that emerge for twists near the "magic angle". Novel predictions include unconventional superconductivity, correlated insulators, anomalous quantum Hall ferromagnetism, and intriguing viscous hydrodynamics.

The physically inspired models lead to far-reaching mathematical questions about limits of quantum particle systems, the modeling of dissipation and electronic viscosity, the topology of energy bands, quasi-periodic Schrödinger operators, semi-classical limits, and the linear and nonlinear optical response, among others. The underlying areas of mathematics span partial differential equations, topology, numerical analysis, and statistical mechanics.

The goal of this interdisciplinary workshop is to bring together mathematicians, and theoretical and applied physicists with the purpose of identifying and discussing new mathematical problems of physical importance in electronic transport.

Organizing committee

SVETLANA JITOMIRSKAYA, UC Irvine

MITCHELL LUSKIN, University of Minnesota

ALLAN H. MACDONALD, University of Texas, Austin

DIONISIOS MARGETIS, University of Maryland

Workshop Schedule

MONDAY, MARCH 11, 2024

- 8:15 - 8:45 BREAKFAST
- 8:45 - 9:00 DORON LEVY (University of Maryland/Director, Brin MRC)
Opening
- 9:00 - 9:45 EFTHIMIOS KAXIRAS (Harvard University)
Twisted Multilayer Graphene Revisited: Where is the Magic?
- 9:45 - 10:15 COFFEE BREAK
- 10:15 - 11:00 ERIC CANCES (CERMICS, Ecole des Ponts ParisTech)
Some Mathematical Results on Graphene and Twisted Bilayer Graphene
- 11:00 - 11:30 PATRICK LEDWITH (Harvard University)
Vortexable Chern Bands and Fractional Chern Insulators in Moire Graphene and Transition Metal Dichalcogenides
- 11:30 - 2:00 LUNCH AT BRIN
- 2:00 - 2:45 ANDREI BERNEVIG (Princeton University)
Topological Heavy Fermion as the Theory for Twisted Bilayer Graphene
- 2:45 - 3:15 COFFEE BREAK
- 3:15 - 4:00 ALEXANDER WATSON (University of Minnesota)
Continuum Limits of Moire Materials

TUESDAY, MARCH 12, 2024

- 8:30 - 9:00 BREAKFAST
- 9:00 - 9:45 TOBIAS STAUBER (Instituto de Ciencia de Materiales de Madrid)
Phase Diagram and Nematic Superconductivity of Twisted Bilayer Graphene
- 9:45 - 10:15 COFFEE BREAK
- 10:15 - 11:00 MICHAEL WEINSTEIN (Columbia University)
Pseudo-magnetism and Landau Levels in Deformed Honeycomb Structures
- 11:00 - 11:30 MICHAEL HOTT (University of Minnesota)
tBG in the Mean-Field: Emergence of Hartree-Fock-Bogoliubov Equations
- 11:30 - 2:00 LUNCH AT BRIN
- 2:00 - 2:45 KEVIN STUBBS (University of California, Berkeley)
Ferromagnetic Ground States in Twisted N-Layer Graphene
- 2:45 - 3:15 COFFEE BREAK
- 3:15 - 3:45 SOLOMON QUINN (University of Minnesota)
Higher-Order Corrections to the Bistritzer-MacDonald Model
- 3:45 - 4:15 TIANYU KONG (University of Minnesota)
Modeling of Electronic Dynamics in Twisted Bilayer Graphene

WEDNESDAY, MARCH 13, 2024

- 8:30 - 9:00 BREAKFAST
- 9:00 - 9:45 DANIEL MASSATT (Louisiana State University)
Momentum Space and Continuum Models of Incommensurate Bilayer 2D Materials
- 9:45 - 10:15 COFFEE BREAK
- 10:15 - 11:00 OSKAR VAPEK (Florida State University)
The effects of strain and out-of-plane magnetic field on electrons in interacting moire materials
- 11:00 - 11:45 ZOE ZHU (Stanford University)
The Role of Electron-Phonon Coupling in the Superconductivity of Twisted Bilayer Graphene
- 11:45 - 2:00 LUNCH AT BRIN
- 2:00 - 2:45 LIANG FU (Massachusetts Institute of Technology)
- 2:45 - 3:15 COFFEE BREAK
- 3:15 - 4:00 GRIGORY TARNOPOLSKY (Carnegie Mellon University)
Magic Angle Butterfly in Twisted Trilayer Graphene
- 4:00 - 4:45 GUILLAUME BAL (University of Chicago)
- 7:00 - 9:00 CONFERENCE DINNER

THURSDAY, MARCH 14, 2024

- 8:30 - 9:00 BREAKFAST
- 9:00 - 9:45 LINGRUI GE (Peking University)
Sharp phase transition for the type I operator
- 9:45 - 10:15 COFFEE BREAK
- 10:15 - 11:00 QI ZHOU (Nankai University)
Phase Transition and Mobility Edges
- 11:00 - 11:45 MATTHIAS MAIER (Texas A&M University)
Spectral Effect of a Magnetic Field on the Electronic Instability of 2D Materials
- 11:45 - 2:00 LUNCH (ON YOUR OWN)
- 1:45 - 2:45 BASSAM FAYAD (University of Maryland)
Reducibility of 1-frequency quasi-periodic cocycles without KAM
- 3:00 - 4:00 SVETLANA JITOMIRSKAYA (University of California, Berkeley)
Dual Lyapunov Exponents and the Robust Ten Martini Problem
- 4:00 - 5:00 HIGH TEA

FRIDAY, MARCH 15, 2024

- 8:30 - 9:00 BREAKFAST
- 9:00 - 9:45 WENCAI LIU (Texas A&M University)
Quasi-periodic in Time Solutions of Nonlinear Random Schrodinger Equations
- 9:45 - 10:15 COFFEE BREAK
- 10:15 - 10:45 MATT POWELL (Georgia Institute of Technology)
Continuity of the Lyapunov Exponent for Gevrey Cocycles
- 10:45 - 11:15 XIAOWEN ZHU (University of Washington)
Edge Spectrum of Topological Insulator with Curved Interface
- 11:15 - 12:00 ILYA KACHKOVSKIY (Michigan State University)
*Anderson Localization for Quasiperiodic Operators with Monotone Potentials:
Perturbative and Non-Perturbative Methods*
- 12:00 - 12:15 WORKSHOP CLOSING
- 12:15 - 2:00 LUNCH (ON YOUR OWN)

Abstracts of talks

Twisted Multilayer Graphene Revisited: Where is the Magic?

EFTHIMIOS KAXIRAS

Harvard University

Monday, March 11, 2024 @ 9:00 AM

We revisit the physics of twisted bilayer and trilayer graphene, from an ab initio perspective. In twisted bilayer graphene (tBLG), the moire pattern observed experimentally clearly shows the formation of different types of domains which can be explained by the atomic relaxation, both in-plane and out-of-plane, using continuum elasticity theory and the Generalized Stacking Fault Energy (GSFE) concept. The relaxation significantly affects the electronic states, leading to a pair of flat bands at the charge neutrality point which are separated by band gaps from the rest. We argue that these features appear for a small range of twist angles, that we call the magic range around 1α . For twisted trilayer graphene (tTLG) the situation is considerably more complicated, with a moire-of-moire pattern emerging for different twist angles. We discuss how the details of the band structure can be crucial for understanding the origin of correlated states and superconductivity in tBLG and tTLG. We also derive a minimal effective tight-binding model that can capture the single-particle physics and explore its implications for correlated electron behavior.

Some Mathematical Results on Graphene and Twisted Bilayer Graphene

ERIC CANCES

CERMICS, Ecole des Ponts ParisTech

Monday, March 11, 2024 @ 10:15 AM

In this talk, I will present some mathematical results on the calculation of conductivity in crystalline materials (especially graphene), and on the derivation of reduced Hamiltonians of Bistritzer-MacDonald type for Twisted Bilayer Graphene using semiclassical analysis.

Vortexable Chern Bands and Fractional Chern Insulators in Moire Graphene and Transition Metal Dichalcogenides

PATRICK LEDWITH

Harvard University

Monday, March 11, 2024 @ 11:00 AM

Fractional Chern insulators realize the remarkable physics of the fractional quantum Hall effect (FQHE) in crystalline systems with Chern bands. The lowest Landau level (LLL) is known to host the FQHE, but not all Chern bands are suitable for realizing fractional Chern insulators (FCI). Previous approaches to stabilizing FCIs focused on mimicking the LLL through momentum space criteria. Here instead we take a real-space perspective by introducing the notion of vortexability. Vortexable Chern bands admit a fixed operator that introduces vortices into any band wavefunction while keeping the state entirely within the same band. Vortexable bands admit trial wavefunctions for FCI states, akin to Laughlin states. In the absence of dispersion and for sufficiently short-ranged interactions, these FCI states are the ground state – independent of the distribution of Berry curvature. Vortexable Chern bands emerge naturally in chiral twisted graphene, and fractional Chern insulators were subsequently observed experimentally. Recently, zero-field fractional Chern insulators, and potentially a zero-field composite Fermi liquid, were also observed in the nearly-vortexable twisted MoTe_2 . New and exciting nearly-vortexable platforms are also appearing, including periodically strained graphene and helically twisted graphene.

Topological Heavy Fermion as the Theory for Twisted Bilayer Graphene

ANDREI BERNEVIG

Princeton University

Monday, March 11, 2024 @ 2:00 PM

I will review the topological heavy fermion model of twisted bilayer graphene and review recent progress in analytical DMFT approximations on this model that can be used to compute transport coefficients. We show calculations that we believe uniquely can match recent Seeback coefficient experiments which show the presence of different carriers with asymmetric mass and asymmetric lifetime. We then show how strain and particle hole breaking relaxation can be easily incorporated into the heavy fermion model which leads to the following results: 1. The explanation of the observation of stronger correlated insulator states on the electron doped side than the hole doped side of tbg, despite the bare dispersion being stronger on that side and 2. An analytical understanding of the stability of the IKS state recently found in STM experiments to be the ground state of the correlated CI including its wavevector. We will also discuss preliminary results on the superconductivity of tbg.

Continuum Limits of Moire Materials

ALEXANDER WATSON

University of Minnesota

Monday, March 11, 2024 @ 3:15 PM

It is generally not practical to study moiré materials through atomic-scale models because, at twist angles close to the magic angle 1° , a moiré unit cell contains thousands of atoms. It is now well-known that the Bistritzer-MacDonald continuum model introduced in 2011 fairly accurately describes the physics of twisted bilayer graphene close to the magic angle. I will present rigorous mathematical work and numerical computations clarifying (1) higher-order corrections to this model and how they affect the model's symmetries, and (2) how the model describes physics even at large twist angles through a homogenization limit.

Phase Diagram and Nematic Superconductivity of Twisted Bilayer Graphene

TOBIAS STAUBER

Instituto de Ciencia de Materiales de Madrid

Tuesday, March 12, 2024 @ 9:00 AM

Typical moire systems consist of 10,000 atoms per unit cell, however, as mainly the emerging flat bands determine the novel correlated and topological phases, an effective density-matrix related to the flat bands usually suffices to describe the main physics. The correlated insulator phases of magic angle bilayer graphene at even integer filling factor, e.g., can be characterized by an $U(4)$ -ferromagnet. But this approximate symmetry is broken in real samples and the way how it is broken may lead to the yet unexplained asymmetry between the superconducting phases for electronic and hole doping.

In order to address the phase diagram of realistic moire systems, I will thus start from a microscopic tight-binding model within the Hartree-Fock approximation and present two ways how to reduce the full density matrix to a density matrix based on a $SU(4)$ -symmetry. We find that at charge neutrality point also the reduced density matrix is described by a pure (valley coherent) state. We further find nematic superconductivity with valley polarization for hole doping and valley coherence for electron doping.

I will finally briefly outline our theory of Ising superconductivity of twisted trilayer graphene and show how real-space chirality can be measured in linear transport experiments.

Pseudo-magnetism and Landau Levels in Deformed Honeycomb Structures

MICHAEL WEINSTEIN

Columbia University

Tuesday, March 12, 2024 @ 10:15 AM

A non-uniform deformation of a honeycomb medium induces effective-magnetic and effective-electric fields. One may choose a deformation which gives rise to a constant perpendicular effective-magnetic field with Landau-level spectrum (flat bands). In the setting of photonic crystals, the tight binding model is generally not applicable. I will present a continuum theory, and some rigorous and formal asymptotic results (joint work with J. Guglielmon and M. Rechtsman - Phys. Rev. A 103 2021), and then review very recent experimental confirmation of this theory (Barsukova et al. <https://arxiv.org/abs/2306.04011>).

tBG in the Mean-Field: Emergence of Hartree-Fock-Bogoliubov Equations

MICHAEL HOTT

University of Minnesota

Tuesday, March 12, 2024 @ 11:00 AM

I will present how to study the validity of the Hartree-Fock-Bogoliubov/Bogoliubov-de Gennes approximation for an interacting Bistritzer-MacDonald model, from the perspective of its quasifree approximation.

Ferromagnetic Ground States in Twisted N-Layer Graphene

KEVIN STUBBS

University of California, Berkeley

Tuesday, March 12, 2024 @ 2:00 PM

Magic angle twisted bilayer graphene exhibits a rich variety of correlated electronic phases, including the correlated insulator phase at integer fillings. Analytic calculations at the chiral limit indicate that certain Slater determinants with a non-zero charge gap can serve as exact many-body ground states for this phase. In a valleyless and spinless model at half filling, two distinct ferromagnetic Slater determinant states can be shown to be exact ground states. We generalize this analysis to twisted N-layer graphene systems. Assuming an additional layer symmetry, we determine conditions so that the ferromagnetic Slater determinants are the only Slater determinant ground states. Through analytical methods, we confirm that these conditions are met in twisted bilayer and equal twist angle trilayer graphene.

Higher-Order Corrections to the Bistritzer-MacDonald Model

SOLOMON QUINN

University of Minnesota

Tuesday, March 12, 2024 @ 3:15 PM

The first-order continuum model proposed by Bistritzer and MacDonald in 2011 accurately describes the electronic properties of twisted bilayer graphene at small twist angles. In this talk, we discuss extensions of the Bistritzer-MacDonald (BM) model to higher-order partial differential equations. We present a derivation of these higher-order models from the tight-binding setting, and show how their solutions can be used to approximate the corresponding tight-binding wave functions. Our main result concerns the error of this approximation, which is small under a natural choice of parameters and given initial conditions that are spectrally localized to the monolayer Dirac points. The higher order corrections indeed improve the accuracy of the continuum model. Moreover, the corrections break symmetries of the original BM model and capture a time-dependent rotation of the wave function not seen at first order. This work builds on the 2023 JMP paper by Watson, Kong, MacDonald, and Luskin, which rigorously established the validity of the (first-order) BM model.

Modeling of Electronic Dynamics in Twisted Bilayer Graphene

TIANYU KONG

University of Minnesota

Tuesday, March 12, 2024 @ 3:45 PM

We consider the problem of numerically computing the quantum dynamics of an electron in twisted bilayer graphene. The challenge is that atomic-scale models of the dynamics are aperiodic for generic twist angles because of the incommensurability of the layers. The Bistritzer-MacDonald PDE model, which is periodic with respect to the bilayer's moire pattern, has recently been shown to rigorously describe these dynamics in a parameter regime. In this talk, we first prove that the dynamics of the tight-binding model of incommensurate twisted bilayer graphene can be approximated by computations on finite domains. The main ingredient of this proof is a speed of propagation estimate proved using Combes-Thomas estimates. We then provide extensive numerical computations which clarify the range of validity of the Bistritzer-MacDonald model.

Momentum Space and Continuum Models of Incommensurate Bilayer 2D Materials

DANIEL MASSATT

Louisiana State University

Wednesday, March 13, 2024 @ 9:00 AM

Electronic structure of incommensurate 2D materials is well approximated by Wannierized tight-binding models, but these models do not immediately realize the critical relationship between momenta and energy vital for applications such as the construction of a many-body basis. Continuum models have become excellent tools for describing the low energy physics such as the Bistritzer-MacDonald model, but much is yet to be understood about the accuracy of these models.

The effects of strain and out-of-plane magnetic field on electrons in interacting moire materials

OSKAR VAPEK

Florida State University

Wednesday, March 13, 2024 @ 10:15 AM

I will discuss the effects of heterostrain and external out of plane magnetic field on the electrons in twisted bilayer graphene. The effects of the electron electron interactions are also considered within the self-consistent Hartree-Fock approximation. In addition, I will discuss effects of the out-of-plane magnetic field on the interacting electrons of twisted MoTe2.

The Role of Electron-Phonon Coupling in the Superconductivity of Twisted Bilayer Graphene

ZOE ZHU

Stanford University

Wednesday, March 13, 2024 @ 11:00 AM

The origin of superconductivity in twisted bilayer graphene (tBLG) has been a topic of heated debate since its discovery. In magic-angle tBLG, the electronic bands significantly flatten, leading to enhanced electronic correlation, a potential precursor to superconductivity. However, the role of electron-phonon coupling has often been overlooked. To accurately describe electron-phonon coupling in moire systems, there are nearly insurmountable challenges related to the incommensurability at general twist angles and the large number of degrees of freedom, including both electron and phonon momenta. In this work, we overcome these computational challenges by employing a first-principles-based multi-scale continuum model to accurately and efficiently describe electron-phonon coupling due to moire phonons in twisted bilayer graphene. We estimate the twist-angle dependence of the superconducting critical temperature without relying on an adiabatic approximation.

TBD

LIANG FU

Massachusetts Institute of Technology

Wednesday, March 13, 2024 @ 2:00 PM

Magic Angle Butterfly in Twisted Trilayer Graphene

GRIGORY TARNOPOLSKY

Carnegie Mellon University

Wednesday, March 13, 2024 @ 3:15 PM

In this talk, I will discuss recent theoretical efforts to understand magic angles in Twisted Trilayer Graphene with two arbitrary twist angles. I'll show that this material hosts a plethora of magic angles, which I'll refer to as the "Magic Angle Butterfly".

TBD

GUILLAUME BAL

University of Chicago

Wednesday, March 13, 2024 @ 4:00 PM

Sharp phase transition for the type I operator

LINGRUI GE

Peking University

Thursday, March 14, 2024 @ 9:00 AM

We will talk about sharp phase transition in frequency for the type I operators, which implies the robustness of the Aubry-Andre-Jitomirskaya phase transition conjecture. This is a joint work with Svetlana Jitomirskaya.

Phase Transition and Mobility Edges

QI ZHOU

Nankai University

Thursday, March 14, 2024 @ 10:15 AM

The disorder systems host three types of fundamental quantum states, known as the extended, localized, and critical states, of which the critical states remain being much less explored. In this talk, we propose several class of exactly solvable models which host exact mobility edges (MEs) separating extended states and localized states, MEs separating localized states from critical states, MEs separating critical states and extended states, we will propose experimental realization and also its rigorous proof.

Spectral Effect of a Magnetic Field on the Electronic Instability of 2D Materials

MATTHIAS MAIER

Texas A&M University

Thursday, March 14, 2024 @ 11:00 AM

In the last two decades, there has been a significant surge in interest surrounding the challenge of effectively producing electromagnetic radiation within the terahertz frequency range. A particularly promising approach lies in leveraging the unique characteristics of two-dimensional materials like graphene and van der Waals heterostructures.

When the size of the sample is large compared to the mean free path for momentum-conserving electron-electron scattering but small compared to the mean free path of momentum-relaxing collisions, the electron system may exhibit fluid-like behavior. This observation gives rise to fluid-dynamical descriptions of electron transport that predict the existence of an unstable subsonic regime leading to the generation of terahertz frequency radiation through unstable 2D electronic transport.

In this talk we discuss how the electronic instability of a two-dimensional inviscid electronic fluid within an elongated channel can be influenced by an external, out-of-plane static magnetic field. As the fluid system approaches the subsonic-to-supersonic transition, our computations reveal a magnetically controlled spectral gap in the imaginary part of the eigen-frequencies. This indicates a non-continuous transition between stability and instability with respect to the effective Mach parameter of the boundary conditions. We conclude by outlining how these findings lead to predictions for the complex spectrum that may challenge physical interpretation.

Joint work with Dionisios Margetis, University of Maryland.

Reducibility of 1-frequency quasi-periodic cocycles without KAM

BASSAM FAYAD

University of Maryland

Thursday, March 14, 2024 @ 1:45 PM

We prove reducibility results for close to constant 1-frequency quasi-periodic matrix valued cocycles in finite and smooth regularity without the use of a quadratic scheme. Reducibility results to rotations' matrices are obtained regardless of the arithmetics of the irrational base frequency. Joint works with Fernando Argenti.

Dual Lyapunov Exponents and the Robust Ten Martini Problem

SVETLANA JITOMIRSKAYA

University of California, Berkeley

Thursday, March 14, 2024 @ 3:00 PM

The Hofstadter butterfly, a plot of the band spectra of almost Mathieu operators at rational frequencies, has become a pictorial symbol of the field of quasiperiodic operators and has gained renewed prominence through experimental study of moire materials. It is visually clear from this plot that for all irrational frequencies the spectrum must be a Cantor set, a statement that has been dubbed the ten martini problem. It has been established for the almost Mathieu operators, exploiting various special features of this family. We will discuss a recently developed robust method allowing to establish it for a large class of one-frequency quasiperiodic operators, including nonperturbative analytic neighborhoods of several popular explicit families. The proof builds on the recently developed concept of dual Lyapunov exponents and partial hyperbolicity of the dual cocycles. Based on joint papers with L. Ge, J. You, and Q. Zhou.

Quasi-periodic in Time Solutions of Nonlinear Random Schrodinger Equations

WENCAI LIU

Texas A&M University

Friday, March 15, 2024 @ 9:00 AM

For Anderson model ($H_\lambda = \Delta + \lambda V$, where Δ is the discrete Laplacian on the lattice \mathbb{Z}^d , $\lambda > 0$ is the strength of the disorder and $V = \{v_n\}_{n \in \mathbb{Z}^d}$ is a family of independent identically distributed random variables), it is known that in one dimension, H_λ has Anderson localization at arbitrary disorder λ and in dimension $d \geq 2$, H_λ has Anderson localization at large disorder λ . With Weimin Wang, we proved a nonlinear version, namely we established the persistence of quasi-periodic in time solutions of nonlinear random Schrödinger equations bifurcating from the linear ones.

Continuity of the Lyapunov Exponent for Gevrey Cocycles

MATT POWELL

Georgia Institute of Technology

Friday, March 15, 2024 @ 10:15 AM

Many spectral properties of 1D Schrödinger operators have been linked to the Lyapunov exponent of the corresponding Schrödinger cocycle. While the situation for one-frequency quasi-periodic operators with analytic potential is well-understood, the multifrequency and non-analytic situation is not. The purpose of this talk is twofold: first, discuss our recent and ongoing work on multi-frequency analytic quasi-periodic cocycles, establishing continuity (both in cocycle and jointly in cocycle and frequency) of the Lyapunov exponent for non-identically singular cocycles, and second, discuss recent work extending these results to suitable Gevrey classes. Analogous results for analytic one-frequency cocycles have been known for over a decade, but the multi-frequency results have been limited to either Diophantine frequencies (continuity in cocycle) or $SL(2, \mathbb{C})$ cocycles (joint continuity). We will discuss the main points of our argument, which extends earlier work of Bourgain.

Edge Spectrum of Topological Insulator with Curved Interface

XIAOWEN ZHU

University of Washington

Friday, March 15, 2024 @ 10:45 AM

Topological insulators (TI) are a class of 2D materials that behave like insulators in their bulk but support robust states along their edges. One of the key properties of TI that is expected to be true is the robustness of the property above w.r.t. the shape of the edge. In this talk, we will discuss how the shape of the edge influences the property of TI above. In particular, we will both give a general, intuitive condition for this property to hold and provide a counter-example otherwise. We also show why in practical situation, experiments may provide misleading results on TI. This work is based on a joint work with Alexis Drouot.

Anderson Localization for Quasiperiodic Operators with Monotone Potentials: Perturbative and Non-Perturbative Methods

ILYA KACHKOVSKIY

Michigan State University

Friday, March 15, 2024 @ 11:15 AM

The general subject of the talk is spectral theory of discrete (tight-binding) Schrodinger operators on d -dimensional lattices. For operators with periodic potentials, it is known that the spectra of such operators are purely absolutely continuous. For random i.i.d. potentials, such as the Anderson model, it is expected and can be proved in many cases that the spectra are almost surely purely point with exponentially decaying eigenfunctions (Anderson localization). Quasiperiodic operators can be placed somewhere in between: depending on the potential sampling function and the Diophantine properties of the frequency and the phase, one can have a large variety of spectral types. We will consider quasiperiodic operators

$$(H(x)\psi)_n = \epsilon(\Delta\psi)_n + f(x + n \cdot \omega)\psi_n, \quad n \in \mathbb{Z}^d,$$

where Δ is the discrete Laplacian, ω is a vector with rationally independent components, and f is a 1-periodic function on \mathbb{R} , monotone on $(0, 1)$ with a positive lower bound on the derivative and some additional regularity properties. We will focus on two methods of proving Anderson localization for such operators: a perturbative method based on direct analysis of cancellations in the Rayleigh-Schrodinger perturbation series for arbitrary d , and a non-perturbative method based on the analysis of Green's functions for $d = 1$, originally developed by S. Jitomirskaya for the almost Mathieu operator. The talk is based on joint works with S. Krymskii, L. Parnovski, and R. Shterenberg (perturbative methods) and S. Jitomirskaya (non-perturbative methods).

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

GUILLAUME BAL, University of Chicago
ANDREI BERNEVIG, Princeton University
ERIC CANCES, CERMICS, Ecole des Ponts ParisTech
BASSAM FAYAD, University of Maryland
LIANG FU, Massachusetts Institute of Technology
LINGRUI GE, Peking University
MICHAEL HOTT, University of Minnesota
SVETLANA JITOMIRSKAYA, University of California, Berkeley
ILYA KACHKOVSKIY, Michigan State University
EFTHIMIOS KAXIRAS, Harvard University
TIANYU KONG, University of Minnesota
PATRICK LEDWITH, Harvard University
DORON LEVY, University of Maryland/Director, Brin MRC
WENCAI LIU, Texas A&M University
MITCHELL LUSKIN, University of Minnesota
MATTHIAS MAIER, Texas A&M University
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