BRIN MATHEMATICS
RESEARCH CENTER

Representation Theory and Lie Groups

- in celebration of the career of Jeffrey Adams

October 14 - 17, 2024

Speakers

Moshe Adrian, Queens College, CUNY
Nicolas Arancibia Robert, CY Cergy Paris Université
Anne-Marie Aubert, Jussieu-Paris Rive Gauche
James Arthur, University of Toronto
Dan Barbasch, Cornell University
Raphaël Beuzart-Plessis, Université d'Aix-Marseille
Dan Ciubotaru, University of Oxford
Dougal Davis, University of Melbourne
Wee Teck Gan, National University of Singapore
Tasho Kaletha, University of Michigan
Lucas Mason-Brown, University of Oxford
Dipendra Prasad, IIT Bombay
Gordan Savin, University of Utah
Birgit Speh, Cornell Univeristy
Marc Van Leeuwen, Université de Poitiers
David Vogan, MIT

About the Workshop

The workshop will focus on recent developments in the theory of representations of Lie groups, in particular, the latest results in the unitary dual problem, advances with the Atlas software, unipotent representations, Arthur packets, and connections with geometric tools such as Hodge theory.

Organizers

Tom Haines, University of Maryland
Paul Mezo, Carleton University
Annegret Paul, Western Michigan University
Yiannis Sakellaridis, Johns Hopkins University

CSIC Building, 4th Floor
8169 Paint Branch Drive
University of Maryland
College Park, MD 20742

BRINMRC.UMD.EDU

DEPARTMENT OF
MATHEMATICS

Contents

Schedule at a Glance

Time	Monday	Tuesday	Wednesday	Thursday
8:00	Breakfast	Breakfast	Breakfast	Breakfast
9:00	Vogan	Aubert	Davis	Beuzart-Plessis
10:00	Coffee Break	Coffee Break	Coffee Break	Coffee Break
11:00	Prasad	Ciubotaru	Mason-Brown	Arthur
12:00	Barbasch	Kaletha	Adrian	
13:00	Lunch	Lunch	Lunch	
14:00	Van Leeuwen	Gan	Speh	
15:00	Coffee Break	Coffee Break	Coffee Break	
16:00	Arancibia Robert	Savin	Excursion	
17:00	Group photo, followed by High Tea			
18:00				

Workshop Overview

The workshop will focus on recent developments in the theory of representations of real Lie groups, in particular, the latest results in the unitary dual problem, the implementation and advances with the Atlas software, unipotent representations, Arthur packets, and connections with geometric tools such as Hodge theory. In addition there will be talks on emerging connections with analogous problems in p-adic groups, in particular related to the construction and behavior of local Langlands and local Arthur packets. We aim to bring together experts in these varied fields to share ideas and methods which should lead to further progress.

Organizing committee

TOM HAINES, University of Maryland

PAUL MEZO, Carleton University

ANNEGRET PAUL Western Michigan University

YIANNIS SAKELLARIDIS, Johns Hopkins University

Workshop Schedule

Monday, October 14, 2024

8:15 - 8:45   Breakfast

8:45 - 9:00   Doron Levy (University of Maryland/Director, Brin MRC)
Opening

9:00 - 9:50   David Vogan (Massachusetts Institute of Technology)
Reductive Group Representations and Weyl Group Representations

10:00 - 10:30   Coffee Break

10:30 - 11:20   Dipendra Prasad (Indian Institute of Technology Bombay)
Some Results on Characters of Compact Connected Lie Groups

11:50 - 12:40   Dan Barbasch (Cornell University)
Some Results on the Shape of the Unitary Dual

12:45 - 2:00   Lunch

2:00 - 2:50   Marc Van Leeuwen (Universite de Poitiers)
Evolution of Atlas, a Tool for Computational Exploration of Representations

3:00 - 3:30   Coffee Break

3:30 - 4:20   Nicolas Arancibia Robert (CY Cergy, Paris Universite)
Computing Arthur Packets with the Atlas Software

4:30 - 5:30   Group photo, followed by High Tea

Tuesday, October 15, 2024

8:30 - 9:00   Breakfast

9:00 - 9:50   Anne-Marie Aubert (Institut de Mathematiques de Jussieu-Paris Rive Gauche)
Bernstein Blocks of Enhanced Langlands Parameters and Applications

10:00 - 10:30   Coffee Break

10:30 - 11:20   Dan Ciubotaru (University of Oxford)
Wavefronts Sets and Langlands Parameters of Smooth Irreducible Representations

11:50 - 12:40   Tasho Kaletha (University of Michigan)
L-Packets and Endoscopy for Discrete Series Parameters of Disconnected Real Reductive Groups

12:45 - 2:00   Lunch

2:00 - 2:50   Wee Teck Gan (National University of Singapore)
Global L-Parameters for G_2

3:00 - 3:30   Coffee Break

3:30 - 4:20   Gordan Savin (University of Utah)
A Family of Exceptional Dual Pairs Where One Member is the Triality D4

Wednesday, October 16, 2024

8:30 - 9:00   Breakfast

9:00 - 9:50   Dougal Davis (The University of Melbourne)
Unitary Representations of Real Groups and Localization Theory for Hodge Modules

10:00 - 10:30   Coffee Break

10:30 - 11:20   Lucas Mason-Brown (University of Oxford)
Arthur Packets and Mixed Hodge Modules

11:50 - 12:40   Moshe Adrian (Queens College, CUNY)
The Sections of the Finite and Affine Weyl Group

12:45 - 2:00   Lunch

2:00 - 2:50   Birgit Speh (Cornell University)
Whitney Extensions on Symmetric Spaces. An Example.

3:00 - 3:30   Coffee Break

3:30 - 5:30   Excursion

5:30 - 5:35   Bus pickup time

6:30 - 9:30   Dinner

Thursday, October 17, 2024

8:30 - 9:00   Breakfast

9:00 - 9:50   Raphael Beuzart-Plessis (Universite d'Aix-Marseille)
Isolation of the Cuspidal Spectrum, Multipliers and the Real Paley-Wiener Theorem

10:00 - 10:30   Coffee Break

10:30 - 11:20   James Arthur (University of Toronto)
Motives and Automorphic Representations

11:30 - 11:45   Workshop Closing

Abstracts of talks

Reductive Group Representations and Weyl Group Representations

David Vogan

Massachusetts Institute of Technology

Monday, October 14, 2024 @ 9:00 AM

Beginning with the Weyl character formula, there has been a long effort to relate the (complicated) representation theory of reductive groups to the (simpler) representation theory of Weyl groups. Jeff Adams has been a leader in this work for forty years. I will explain one way to attach a Weyl group representation to a representation of a real reductive group; recall how Jeff taught the atlas software to compute it; and say a little about connections with endoscopic transfer (another old favorite topic of his).

Some Results on Characters of Compact Connected Lie Groups

Dipendra Prasad

Indian Institute of Technology Bombay

Monday, October 14, 2024 @ 10:30 AM

The Weyl character formula, in principle, tells everything that one could possibly care to ask about character theory of compact connected Lie groups. However, in practice, this is often not easy. In particular, it appears that the character values at specific elements have not been calculated, such as at elements of order 2, or at other torsion elements, or at other special elements, which often have beautiful structure. The lecture will discuss some of these matters, mostly due to others.

Some Results on the Shape of the Unitary Dual

Dan Barbasch

Cornell University

Monday, October 14, 2024 @ 11:50 AM

In this talk I will review older results (joint with Adams) and newer results on the shape of complementary series for real and complex groups.

Evolution of Atlas, a Tool for Computational Exploration of Representations

Marc Van Leeuwen

Universite de Poitiers

Monday, October 14, 2024 @ 2:00 PM

Atlas is a program for the computational exploration of representations of real Lie groups, created by Fokko du Cloux in 2004, and developed in the context of the international project Atlas of Lie Groups and Representations (led by Jeff Adams) until the current day. From a program that initially aimed mainly at producing tables of Kazhdan-Lusztig-Vogan polynomials, it has grown out to a tool for exploring the unitary dual of real Lie groups. In the process it has also shown to be a useful platform for many related computations, such as representations of Weyl groups and the study of nilpotent orbits.

In this talk, we discuss aspects of the design and implementation of the program, with an emphasis on the user interface for commands and programming, rather than on the deeper mathematical notions and algorithms at the program's core. We shall discuss and illustrate with examples, the design philosophy and decisions made. These concern such things as the user interaction model, characteristics of the provided language and its type system, as well as the impact of those decisions on the practical use and usability of the program. We shall contrast and try to compare these to alternative approaches that could have been chosen in the past, and touch on challenges that could be encountered in future development.

Computing Arthur Packets with the Atlas Software

Nicolas Arancibia Robert

CY Cergy, Paris Universite

Monday, October 14, 2024 @ 3:30 PM

In his work on the classification of discrete automorphic spectra, J. Arthur gives a definition of A-packets for quasisplit symplectic and special orthogonal groups. Guided by Arthur's ideas, C.P Mok extends his definition to unitary groups. We explain how to implement Arthur and Mok's construction using the Atlas software. More precisely, in a recent joint work with J. Adams and P. Mezo we were able to implement an algorithm that computes A-packets for pure real forms of symplectic, orthogonal and unitary groups. Atlas now has a function which, upon entering an A-parameter as input, gives the corresponding A-packet as output. We present the algorithm, explain how to use it and compute some examples.

Bernstein Blocks of Enhanced Langlands Parameters and Applications

Anne-Marie Aubert

Institut de Mathematiques de Jussieu-Paris Rive Gauche

Tuesday, October 15, 2024 @ 9:00 AM

In the first part of the talk we will focus on the category of H-equivariant perverse sheaves on the unipotent variety of a (possibly disconnected) complex reductive group H.
Next, we will input these results of geometrical nature into the study of enhanced Langlands parameters of inner twists of p-adic groups and the construction of the local Langlands correspondence, with an emphasis on unipotent representations of p-adic groups. We will also provide applications."

Wavefronts Sets and Langlands Parameters of Smooth Irreducible Representations

Dan Ciubotaru

University of Oxford

Tuesday, October 15, 2024 @ 10:30 AM

The wavefront set of the Harish-Chandra distribution character of irreducible smooth representation is a fundamental invariant and therefore, it is natural to ask how it relates with the local Langlands correspondence. In the setting of irreducible (g,K)-modules of a real reductive Lie group, the wavefront set admits a uniform description in terms of the singular support of the perverse sheaf parameter in the formulation of the Langlands correspondence due to Adams, Barbasch, and Vogan. For a large class of unipotent representations of p-adic groups, although the methods are very different, the analogous description holds, as proven, in joint work with Lucas Mason-Brown and Emile Okada. However, beyond the unipotent case, the relation between the wavefront sets of p-adic group representations and the Langlands parameter is more complicated. I will present what is known and give evidence for a conjectural upper bound of the Langlands parameter, as formulated in joint work with Ju-Lee Kim.

L-Packets and Endoscopy for Discrete Series Parameters of Disconnected Real Reductive Groups

Tasho Kaletha

University of Michigan

Tuesday, October 15, 2024 @ 11:50 AM

The construction of L-packets and the proof of their ordinary endoscopic character identities for connected real reductive groups are celebrated results of Langlands and Shelstad. In a recent preprint we proposed conjectural extensions of these results to the setting of disconnected reductive groups over local fields. We will discuss the proof of these conjectures when the local field is the real numbers, and the L-parameter is discrete. This is joint work with Paul Mezo.

Global L-Parameters for G_2

WEE TECK GAN

National University of Singapore

Tuesday, October 15, 2024 @ 2:00 PM

Globally generic cuspidal representations of G_2 are known to have functorial lifts to $GL(7)$. In a joint work with Erez Lapid, we determine precisely the image of this lift. This provides one with a notion of global L-parameters for G_2.

A Family of Exceptional Dual Pairs Where One Member is the Triality D4

GORDAN SAVIN

University of Utah

Tuesday, October 15, 2024 @ 3:30 PM

I will describe the D4 family of dual pairs in exceptional groups, and discuss in details the case when the ambient group is exceptional E6. In that case, a complete description of global and p-adic correspondence was obtained in a work with Gan, followed by a work with Gan, Loke and Paul, where the case of real groups was treated with a help from Atlas software. An overview of these results will be given, with emphasis on results for real groups.

Unitary Representations of Real Groups and Localization Theory for Hodge Modules

DOUGAL DAVIS

The University of Melbourne

Wednesday, October 16, 2024 @ 9:00 AM

I will discuss recent joint work with Kari Vilonen, in which we prove a conjecture of Schmid-Vilonen linking unitary representations of real reductive groups with Hodge theory. Our main result is that the unitarity of an irreducible representation can be read off from a canonical filtration, the Hodge filtration, defined using Beilinson-Bernstein localization and the geometry of the complex flag variety. Our proof follows a strategy proposed by Adams, Trapa and Vogan, the core idea of which is to show that the Atlas algorithm computing the signatures of Hermitian forms also computes the degrees of the Hodge filtration. If time permits, I will also discuss how excellent homological properties of the Hodge filtration lead to a general proof of unitarity for a wide class of unipotent representations, to appear in joint work with Lucas Mason-Brown.

Arthur Packets and Mixed Hodge Modules

LUCAS MASON-BROWN

University of Oxford

Wednesday, October 16, 2024 @ 10:30 AM

Let G be a real reductive group. In the 1990s, Adams, Barbasch, and Vogan defined some finite sets of irreducible G-representations called "Arthur packets". It is conjectured that all Arthur packets consist of unitary representations. In this talk, I will explain a strategy for proving this conjecture in general, using Hodge-theoretic techniques. This is based on joint work with Dougal Davis, Jeff Adams, Andrei Ionov, and David Vogan.

The Sections of the Finite and Affine Weyl Group

Moshe Adrian

Queens College, CUNY

Wednesday, October 16, 2024 @ 11:50 AM

Let G be a split, connected reductive group over an algebraically closed field F. In 1966, Tits defined a section of the finite Weyl group W which satisfies the braid relations. This section has been used extensively in various works. Motivated by a question about the Kottwitz homomorphism, we determine all sections of W that satisfy the braid relations, in the case that G is almost-simple. We will then discuss the structure of these sections, as well as applications.

Now let F be p-adic. In 2015, Ganapathy extended Tits' section to a section of the affine Weyl group W^a, satisfying the braid relations. We determine all sections of W^a that satisfy the braid relations, and again discuss their structure and applications.

Finally, if there is time, we will discuss the same topics for the extended affine Weyl group W^e.

Whitney Extensions on Symmetric Spaces. An Example.

Birgit Speh

Cornell University

Wednesday, October 16, 2024 @ 2:00 PM

In 1932 H.Whitney introduced the problem of extending a function defined on a set of points to an differentiable or even analytic function on the ambient space. I will discuss this problem for a symmetric space X(p,q) =SO(p,q)/SO(p-1,q) using harmonic analysis on the space. This is joint work with Peter Uttenthal.

Isolation of the Cuspidal Spectrum, Multipliers and the Real Paley-Wiener Theorem

RAPHAEL BEUZART-PLESSIS

Universite d'Aix-Marseille

Thursday, October 17, 2024 @ 9:00 AM

In this talk, I will review a construction of convolution operators that isolate certain cuspidal representations from the rest of the automorphic spectrum. This allows for drastic simplifications in the analysis of spectral sides of certain (relative) trace formulas. For this, we combine the action of spherical Hecke algebras at unramified places with that of an algebra of multipliers at Archimedean places. In particular, it is crucial that the multiplier algebra we use be sufficiently large. Time permitting, I will also speculate on a possible extension of Arthur's Paley-Wiener theorem for real reductive groups that fits very naturally in this story. This is based on joint work with Yifeng Liu, Wei Zhang and Xinwen Zhu.

Motives and Automorphic Representations

JAMES ARTHUR

University of Toronto

Thursday, October 17, 2024 @ 10:30 AM

We shall describe a conjectural concrete formula for the (pure) motivic Galois group. It would be a consequence of a parallel formula for an automorphic Galois group based on the principle of functoriality, which would classify automorphic representations. If time permits, we would add remarks on the possible extension of the motivic Galois group to a larger group that would include mixed motives, and possibly even exponential motives.

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

JEFF ADAMS, University of Maryland
JEFFREY ADLER, American University
MOSHE ADRIAN, Queens College, CUNY
SPYRIDON AFENTOULIDIS ALMPANIS, Bar-Ilan University
NICOLAS ARANCIBIA ROBERT, CY Cergy, Paris Universite
JAMES ARTHUR, University of Toronto
MAHDI ASGARI, Oklahoma State University
ANNE-MARIE AUBERT, Institut de Mathematiques de Jussieu-Paris Rive Gauche
DAN BARBASCH, Cornell University
LETICIA BARCHINI, Oklahoma State University
ANSUMAN BARDALAI, University of California, Berkeley
RAPHAEL BEUZART-PLESSIS, Universite d'Aix-Marseille
DAN CIUBOTARU, University of Oxford
DOUGAL DAVIS, The University of Melbourne
PETER DILLERY, University of Maryland
CHENGZE DUAN, University of Massachusetts-Amherst
RAM EKSTROM, University of Michigan
XANDER FABER, Institute for Defense Analysis
ANDREW FROHMADER, University of Wisconsin - Milwaukee
WEE TECK GAN, National University of Singapore
THOMAS HAINES, University of Maryland
ALEXANDER HAZELTINE, University of Michigan
TASHO KALETHA, University of Michigan
DORON LEVY, University of Maryland/Director, Brin MRC
RON LIPSMAN, University of Maryland
CHI-HENG LO, Purdue University
DAVID MANDERSCHEID, National Science Foundation
LUCAS MASON-BROWN, University of Oxford
PAUL MEZO, Carleton University
EMILE OKADA, National University of Singapore
BASUDEV PATTANAYAK, The University of Hong Kong
ANNEGRET PAUL, Western Michigan University
DIPENDRA PRASAD, Indian Institute of Technology Bombay
RAVI RAGHUNATHAN, Indian Institute of Technology
MISHTY RAY, University of Calgary

YIANNIS SAKELLARIDIS, Johns Hopkins University
GORDAN SAVIN, University of Utah
BIRGIT SPEH, Cornell University
JIM STANKEWICZ , IDA Center for Computing Sciences
WAN-YU TSAI, National Central University, Taiwan
MARC VAN LEEUWEN, Universite de Poitiers
KARI VILONEN, The University of Melbourne
DAVID VOGAN, Massachusetts Institute of Technology
HONGFENG ZHANG, The University of Hong Kong
JIALIANG ZOU, University of Michigan