

# Dynamics Students Days 2024

April 1-4, 2024



## About the Meeting

A meeting for students working in all areas of Dynamical Systems to present and discuss their research, among themselves and with the advisors who attend the meeting.

## Organizers

Dmitry Dolgopyat, University of Maryland  
Spencer Durham, University of Maryland  
Bassam Fayad, University of Maryland  
Enrique Pujals, City University of New York



## Speakers

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Max Auer, University of Maryland  
Lael Costa, Penn State University  
Caleb Dilsavor, Ohio State University  
Spencer Durham, University of Maryland  
Grisha Dvorkin, University of Chicago  
Rose Elliot Smith, University of Central Florida  
Giovanni Forni, University of Maryland  
Bryce Gollobit, City University of New York  
Axel Kodat, City University of New York

Piotr Laskawiec, Penn State University  
Zaicun Li, University of Maryland  
Vaughn Osterman, University of Maryland  
Joshua Paik, Penn State University  
James Reber, Ohio State University  
Megan Roda, University of Chicago  
Anastasiia Sharipova, Penn State University  
Shuo Yan, University of Maryland



DEPARTMENT OF  
MATHEMATICS

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BRIN MATHEMATICS  
RESEARCH CENTER



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# Schedule at a Glance

	Monday	Tuesday	Wednesday	Thursday
9:00				
10:00	Breakfast Laskawiec	Breakfast Roda	Breakfast Sharipova	Breakfast Dvorokin
11:00				
12:00	Reber	Elliott Smith	Costa	Gollobit
13:00	Lunch	Lunch	Lunch	Lunch
14:00	Kodat	Durham	Li	
15:00			Paik	
	Coffee Break	Coffee Break		
16:00	Dilsavor	Auer	Discussion	
			Forni	
17:00	Osterman	Yan	High Tea	
18:00				

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# Workshop Overview

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A meeting for students working in all areas of Dynamical Systems to present and discuss their research, among themselves and with the advisors who attend the meeting.

## Organizing committee

DMITRY DOLGOPYAT, University of Maryland

SPENCER DURHAM, University of Maryland

BASSAM FAYAD, University of Maryland

ENRIQUE PUJALS , City University of New York

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# Workshop Schedule

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**MONDAY, APRIL 1, 2024**

9:30 - 10:00 BREAKFAST

10:00 - 10:50 PIOTR LASKAWIEC (Penn State University)  
*Spectrum Maximizing Products of pairs of  $2 \times 2$  matrices*

10:50 - 11:10 DISCUSSION

11:10 - 12:00 JAMES REBER (Ohio State University)  
*A Magnetic Hopf Theorem*

12:00 - 12:20 DISCUSSION

12:30 - 2:00 LUNCH

2:00 - 2:50 AXEL KODAT (City University of New York)  
*An average intersection estimate for families of diffeomorphisms*

2:50 - 3:10 DISCUSSION

3:10 - 3:40 COFFEE BREAK

3:40 - 4:30 CALEB DILSAVOR (Ohio State University)  
*Holder continuity in metric Anosov flows*

4:30 - 5:20 VAUGHN OSTERMAN (University of Maryland)  
*Elliptic Islands in the Planar Circular Restricted 3-Body Problem*

## TUESDAY, APRIL 2, 2024

9:30 - 10:00 BREAKFAST

10:00 - 10:50 MEGAN RODA (University of Chicago)  
*Averaging Invariant Measures*

10:50 - 11:10 DISCUSSION

11:10 - 12:00 ROSE ELLIOT SMITH (University of Chicago)  
*On measure rigidity of  $u$ -Gibbs states*

12:00 - 12:20 DISCUSSION

12:30 - 2:00 LUNCH

2:00 - 2:50 SPENCER DURHAM (University of Maryland)  
*High-rank rigidity and stability of the cohomological equation*

2:50 - 3:10 DISCUSSION

3:10 - 3:40 COFFEE BREAK

3:40 - 4:30 MAX AUER (University of Maryland)  
*Time changes of weakly mixing flows*

4:30 - 5:20 SHUO YAN (University of Maryland)  
*Energy growth for systems of coupled oscillators with partial damping*

## WEDNESDAY, APRIL 3, 2024

9:30 - 10:00 BREAKFAST

10:00 - 10:50 ANASTASIIA SHARIPOVA (Penn State University)  
*Ivrii Conjecture for some cases in outer and symplectic billiards*

10:50 - 11:10 DISCUSSION

11:10 - 12:00 LAEL COSTA (Penn State University)  
*Outer billiards*

12:00 - 1:30 LUNCH

1:30 - 2:20 ZAICUN LI (University of Maryland)  
*KAM stability for outer billiards*

2:20 - 2:40 DISCUSSION

2:40 - 3:30 JOSHUA PAIK (Penn State University)  
*Averaging invariant measures*

3:30 - 4:00 DISCUSSION

4:00 - 5:00 GIOVANNI FORNI (University of Maryland)  
*Rational polygonal billiards and Teichmueller dynamics*

5:00 - 6:00 HIGH TEA



## THURSDAY, APRIL 4, 2024

9:30 - 10:00 BREAKFAST

10:00 - 10:50 GRISHA DVORKIN (Penn State University)  
*Evolution of a small spheres in Anosov and other smooth systems*

10:50 - 11:10 DISCUSSION

11:10 - 12:00 BRYCE GOLLOBIT (City University of New York)  
*Tangent bundle dynamics*

12:00 - 1:30 LUNCH

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# Abstracts of talks

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## Spectrum Maximizing Products of pairs of 2x2 matrices

PIOTR LASKAWIEC

*Penn State University*

Monday, April 1, 2024 @ 10:00 AM

A spectrum maximizing product (SMP) is a product of matrices whose normalized spectral radius is equal to the joint spectral radius. There are many conjectures regarding the behavior of SMP's. In this talk we will discuss why, in the case of 2x2 matrices, SMP's are not generically unique, contrary to a previously posed conjecture. We will also discuss a partial classification of SMPs. We will see that for the case of pairs of matrices that have distinct real eigenvalues, there is a lot that can be said about what products can and cannot be SMPs. Joint work with Jairo Bochi.

## A Magnetic Hopf Theorem

JAMES REBER

*Ohio State University*

Monday, April 1, 2024 @ 11:10 AM

It is well known that if the sectional curvature of a closed manifold is non-positive, then the associated geodesic flow is without conjugate points. While the converse of this is not true in general, E. Hopf was able to show that if a geodesic flow on a closed surface is without conjugate points, then the total curvature is non-positive (this would later be generalized by L. Green to higher dimensions). In this talk, we discuss a version of this for magnetic systems, which can be thought of as geodesic flows twisted by a closed 2-form. This is joint work with Valerio Assenza and Ivo Terek.

# An average intersection estimate for families of diffeomorphisms

AXEL KODAT

*City University of New York*

Monday, April 1, 2024 @ 2:00 PM

Poincaré's formula in integral geometry states that for any submanifolds of  $V$ ,  $W$  of a compact homogeneous space  $M = G/H$  with  $G$  acting transitively on tangent planes, the average volume of the intersection between  $g(V)$  and  $W$  is equal to  $C \text{vol}(V) \text{vol}(W)$  for  $C$  a universal constant depending only on the dimensions of  $V$  and  $W$ . We discuss an adaptation of this result to general (non-homogeneous) closed manifolds, where the transformation group  $G$  is replaced by a compact family of diffeomorphisms, and the formula now holds up to uniform multiplicative error. We also sketch some possible applications of this result to the dynamics of diffeomorphisms with exponential volume growth.

# Holder continuity in metric Anosov flows

CALEB DILSAVOR

*Ohio State University*

Monday, April 1, 2024 @ 3:40 PM

Under mild assumptions, we show that the local product structure of a (not necessarily compact) metric Anosov flow must be Holder continuous. These assumptions are satisfied by the geodesic flow of a CAT(-1) space when it is equipped with a standard metric, or by the geodesic flow of a pinched negatively curved manifold equipped with the Sasaki metric. This includes the example constructed by Ballmann, Brin, and Burns of a pinched negatively curved surface whose strong stable foliation is not Holder continuous in the distributional sense. In particular, the holonomies between the transverse foliations  $W^s$ s and  $W^u$  of the Ballmann-Brin-Burns example are Holder continuous. This is surprising in conjunction with a result of Hasselblatt and Wilkinson which states that Holder continuity of holonomies (in a stronger sense) implies Holder continuity in the distributional sense.

# Elliptic Islands in the Planar Circular Restricted 3-Body Problem

VAUGHN OSTERMAN

*University of Maryland*

Monday, April 1, 2024 @ 4:30 PM

We consider the planar circular restricted three body problem, modeling the motion of a massless asteroid in the plane undergoing gravitational attraction toward two bodies, each with a circular orbit around their center of mass. For small mass ratios, this is approximated by the Kepler problem as long as the asteroid remains far from the smaller body. The existence of hyperbolic sets containing orbits in which the asteroid undergoes repeated close interactions with the smaller body was proven independently by Bolotin and MacKay and by Font, Nunes, and Simo. My result is that there are elliptic periodic orbits with repeated close interactions. In these orbits, the asteroid remains close to a Kepler orbit which intersects the orbit of the smaller body.

## Averaging Invariant Measures

MEGAN RODA

*University of Chicago*

Tuesday, April 2, 2024 @ 10:00 AM

Let  $X$  be a K3 surface with a large automorphism group  $\text{Aut}(X)$  (we do not assume that it contains any parabolic elements). Consider a probability measure  $\nu$  on  $\text{Aut}(X)$ ; using the work of Cantat and DuJardin (2020) we study hyperbolic, ergodic  $\nu$ -stationary probability measures, and the supports of their conditional measures on the stable and unstable manifolds (which are a.e. biholomorphic to  $\mathbb{C}$ ) using the techniques of Benoist and Quint (2011), and Eskin and Mirzakhani (2018).

# On measure rigidity of u-Gibbs states

ROSE ELLIOT SMITH

*University of Chicago*

Tuesday, April 2, 2024 @ 11:10 AM

Physical measures are an important tool in the study of hyperbolic dynamics, governing, for example, the statistical properties of the orbit of almost every point with respect to volume (in the dissipative setting). The well-studied uniformly hyperbolic (Anosov) diffeomorphisms and flows always have ergodic physical measures, whereas the more general class of partially hyperbolic systems lose this property. For these systems, we are instead guaranteed the existence of at least one, and possibly infinitely many, ergodic u-Gibbs measure(s). In the case of a unique u-Gibbs measure, that measure is automatically physical.

Thus, a natural question in the partially hyperbolic setting is the following: under what conditions is there a unique u-Gibbs measure? More generally, which u-Gibbs measures are physical? This question was partially answered in dimension three by Eskin, Potrie, and Zhang. Here we partially extend this result to arbitrary dimensions, and discuss the dichotomy that arises: roughly, a u-Gibbs measure is physical if and only if it is not jointly integrable of some order.

## High-rank rigidity and stability of the cohomological equation

SPENCER DURHAM

*University of Maryland*

Tuesday, April 2, 2024 @ 2:00 PM

While rigidity is rare in the case of  $Z$ -actions, it is much more common for  $Z^k$  actions for  $k \geq 2$ . I will discuss some of the standard rigidity results for elliptic and partially hyperbolic actions before turning attention to the parabolic case. In particular, I will state a theorem and introduce some techniques used by Damjanovich, Fayad, and Sarpykina (2023) before showing how similar techniques can be used to study the regularity of solutions to the cohomological equation over parabolic actions. This is joint work with Bassam Fayad.

# Time changes of weakly mixing flows

MAX AUER

*University of Maryland*

Tuesday, April 2, 2024 @ 3:40 PM

Minimal linear flows on  $T^2$  and their time changes are one of the most fundamental examples of parabolic dynamical systems. We shall be interested in statistical properties, in particular weak mixing. It is known that a generic smooth time change results in a weakly mixing flow, we provide an example where the speed of weak mixing can be shown to be polynomial. This also provides an example of a  $C_k$  diffeomorphism that is polynomial weak mixing, but not mixing.

# Energy growth for systems of coupled oscillators with partial damping

SHUO YAN

*University of Maryland*

Tuesday, April 2, 2024 @ 4:30 PM

We consider two interacting particles on the circle. The particles are subject to stochastic forcing, which is modeled by white noise. In addition, one of the particles is subject to friction, which models energy dissipation due to the interaction with the environment. We show that, in the diffusive limit, the absolute value of the velocity of the other particle converges to the reflected Brownian motion. In other words, the interaction between the particles are asymptotically negligible in the scaling limit. The proof combines averaging for large energies with large deviation estimates for small energies.

# Ivrii Conjecture for some cases in outer and symplectic billiards

ANASTASIIA SHARIPOVA

*Penn State University*

Wednesday, April 3, 2024 @ 10:00 AM

I give a proof for  $(2n + 1, n)$  and  $(2n, n - 1)$ -periodic Ivrii conjecture for planar outer billiards. I also give new simple geometric proofs for the 3 and 4-periodic cases for outer and symplectic billiards, and generalize for higher dimensions in case of symplectic billiards.

## Outer billiards

LAEL COSTA

*Penn State University*

Wednesday, April 3, 2024 @ 11:10 AM

This talk will introduce the notion of outer or dual billiards to those unfamiliar, give an overview of historical and recent results, and demonstrate outer billiards and its variants with interactive software.

# KAM stability for outer billiards

ZAICUN LI

*University of Maryland*

Wednesday, April 3, 2024 @ 1:30 PM

Since Moser used outer billiard systems as simple models to illustrate KAM theory and study stability in the n-body problem many progress have been made in this field. It is well known that for billiard maps of strictly convex  $C^6$  curves all orbits are bounded. However, when the smoothness condition is absent, the global KAM theory breaks down and diffusion appears. This means escaping orbits are possible in the outer billiard systems when the convex closed curve is not smooth. Moser uses piecewise smooth convex curves as a toy model to explore these phenomena, and the half disc is one of the simplest cases. Dmitry Dolgopyat and Bassam Fayad, in their work in 2009, successfully carried out this question by showing that there is an open ball escaping to infinity in semi-disc outer billiards.

In our work, the same semi-disc outer billiard is considered, and the same domain and coordinates are used. Although global KAM breaks down, we can detect stability locally. Using Birkhoff normal form and KAM theory, we prove that there are infinitely many elliptic fixed points with stable neighborhoods near them and these neighborhoods will occupy a positive proportion of the plane.

## Averaging invariant measures

JOSHUA PAIK

*Penn State University*

Wednesday, April 3, 2024 @ 2:40 PM

Suppose we are given a family of dynamical systems  $F$  acting on a compact metric space  $X$ . When is it possible to disintegrate the uniform distribution on  $X$ , with respect to  $F$ ? More concretely, is there a way to pick an  $f$  invariant measure, for every  $f$ , so that the average of these invariant measures is the uniform distribution? We restrict our study to when  $F$  is a coset of a compact group. In this situation, if we can pick such "magic measures", we call the collection of all of these a Dedieu-Shub family. In this talk, I will detail some examples of Dedieu-Shub measures and applications. This is joint work with Jairo Bochi.



# Rational polygonal billiards and Teichmueller dynamics

GIOVANNI FORNI

*University of Maryland*

Wednesday, April 3, 2024 @ 4:00 PM

# Evolution of a small spheres in Anosov and other smooth systems

GRISHA DVORKIN

*Penn State University*

Thursday, April 4, 2024 @ 10:00 AM

Following B. Gurevich approach we will study the measure of small neighbourhoods of images of small spheres under an action of some diffeos and flows, which somehow characterize distortion of boundaries of the spheres and strongly connected with Lyapunov exponents and metric entropy.

# Tangent bundle dynamics

BRYCE GOLLOBIT

*City University of New York*

Thursday, April 4, 2024 @ 11:10 AM

We'll discuss the dynamics of the pushforward of  $f \in \text{Diff}^1(M)$  and its relationship to the dynamics of  $f$ .

The pushforward,  $T_f$  is an infinite dimensional linear map acting on the space of continuous vector fields of the tangent bundle.

We introduce new invariant subspaces associated to any linear map of a Banach space, based on chains -for the pushforward, these subspaces are the spaces of vector fields a semi-continuous bundle.

Examples include characterizations of Axiom A systems by the dynamics of the pushforward - we'll pay special attention to those satisfying the transversality condition and to quasi-Anosov systems.

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# The Brin Mathematics Research Center

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The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

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# List of Participants

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MAX AUER, University of Maryland  
MARGARET BROWN, Penn State University  
RASIEL CHISHTI, Penn State University  
LAEL COSTA, Penn State University  
JASON DAY, University of Houston  
CALEB DILSAVOR, Ohio State University  
EMMA DINOWITZ, City University of New York  
DMITRY DOLGOPYAT, University of Maryland  
SPENCER DURHAM, University of Maryland  
GRISHA DVORKIN, Penn State University  
ROSE ELLIOT SMITH, University of Chicago  
BASSAM FAYAD, University of Maryland  
GIOVANNI FORNI, University of Maryland  
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XIAYIMEI HAN, University of Maryland  
NOLAN KING, Penn State University  
AXEL KODAT, City University of New York  
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MEGAN RODA, University of Chicago  
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ATA TAHURI, Penn State University  
KUMARI TEENA, University of Houston  
SHUO YAN, University of Maryland