BRIN MATHEMATICS RESEARCH CENTER

Vistas in Number Theory

June 3-7, 2024

About the Workshop

The goal of this conference is to bring together leading researchers who might not routinely interact with each other with the hope of sparking interesting discussions and projects leading to newcollaborations.

Organizers

Niranjan Ramachandran, University of Maryland
Larry Washington, University of Maryland

Participants

Jeff Achter, Colorado State University
Kenichi Bannai, Keio University
James Borger, Australian National University
Bryden Cais, University of Arizona
Pete Clark, University of Georgia
Ralph Greenberg, University of Washington
David Hubbard (Retired)
Yoshi-Hiro Ishikawa, Okayama University
Hershy Kisilevsky, Concordia University
Shinichi Kobayashi, Kyushu University
Angel Kumchev, Towson University
Debanjana Kundu, UT Rio Grande Valley
Antonio Lei, University of Ottawa, Canada
Carlo Pagano, Concordia University
Matt Papanikolas, Texas A&M University
James Parson, Hood College
Cristian Popescu, UC San Diego
Jonathan Sands, University of Vermont
Sankar Sitaraman, Howard University
Johannes Sprang, University of Duisburg-Essen
Padmavathi Srinivasan, Boston University
Florian Sprung, Arizona State University
Jiuya Wang, University of Georgia
C S Yogananda, Chanakya University
David Zywina, Cornell University

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DEPARTMENT OF MATHEMATICS

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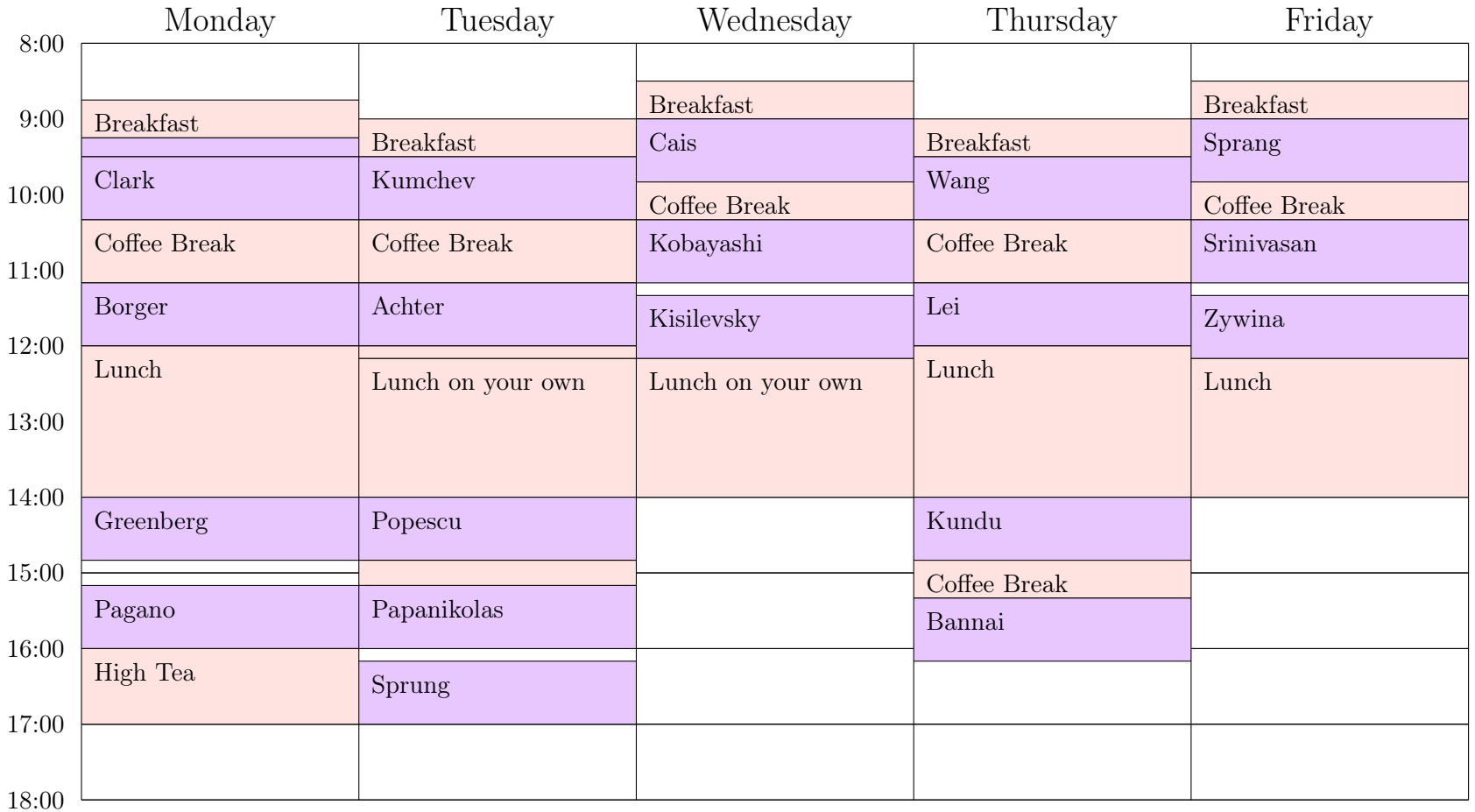
Schedule at a Glance



Workshop Overview

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Organizing committee

Niranjan Ramachandran, University of Maryland

Larry Washington, University of Maryland

Workshop Schedule

Monday, June 3, 2024

8:45 - 9:15	Breakfast
9:15 - 9:30	Doron Levy (University of Maryland/Director, Brin MRC) *Opening*
9:30 - 10:20	Pete Clark (University of Georgia) *Group Theoretic Ax-Katz Theorems*
10:20 - 11:10	Coffee Break
11:10 - 12:00	James Borger (Australian National University) *Scheme theory over semirings and narrow class groups*
12:00 - 2:00	Lunch
2:00 - 2:50	Ralph Greenberg (University of Washington) *Pseudo-null Modules in Iwasawa Theory*
3:10 - 4:00	Carlo Pagano (Concordia University) *Distribution of Selmer groups in quadratic twist families*
4:00 - 5:00	High Tea

Tuesday, June 4, 2024

9:00 - 9:30 Breakfast

9:30 - 10:20 Angel Kumchev (Towson University)
Large Gaps Between k-free Integers and Polynomial Analogs

10:20 - 11:10 Coffee Break

11:10 - 12:00 Jeff Achter (Colorado State University)
Torsion finite problems

12:00 - 12:10 Group Photo

12:10 - 2:00 Lunch on your own

2:00 - 2:50 Cristian Popescu (University of California, San Diego)
An equivariant Tamagawa number formula for abelian t–motives and applications

2:50 - 3:10 Coffee Break

3:10 - 4:00 Matt Papanikolas (Texas A&M University)
Hecke L-series for Sinha modules

4:10 - 5:00 Florian Sprung (Arizona State University)
Hilbert's Tenth Problem for Some New Families of Number Fields

Wednesday, June 5, 2024

8:30 - 9:00 Breakfast

9:00 - 9:50 Bryden Cais (University of Arizona)
Iwasawa-theoretic analogues of the Riemann–Hurwitz and Deuring–Shafarevich formulae

9:50 - 10:20 Coffee Break

10:20 - 11:10 Shinichi Kobayashi (Kyushu University)
The p-adic valuation of local resolvents and anticyclotomic Hecke L-values of imaginary quadratic fields at inert prime

11:20 - 12:10 Hershy Kisilevsky (Concordia University)
Non-zero Central Values of Dirichlet Twists of Elliptic L-Functions

12:10 - 2:00 Lunch on your own

Thursday, June 6, 2024

9:00 - 9:30 Breakfast

9:30 - 10:20 Jiuya Wang (University of Georgia)
Dominant Galois Groups for Large Degree Number Fields

10:20 - 11:10 Coffee Break

11:10 - 12:00 Antonio Lei (University of Ottawa)
Iwasawa theory of isogeny graphs with level structures

12:00 - 2:00 Lunch

2:00 - 2:50 Debanjana Kundu (University of Texas - Rio Grande Valley)
Iwasawa Theory and Arithmetic Statistics

2:50 - 3:20 Coffee Break

3:20 - 4:10 Kenichi Bannai (Keio University)
On the Algebraic Torus associated to Totally Real Fields and the Shintani Generating Class

Friday, June 7, 2024

8:30 - 9:00 Breakfast

9:00 - 9:50 Johannes Sprang (University of Duisburg-Essen)
Irrationality and linear independence of p-adic zeta values

9:50 - 10:20 Coffee Break

10:20 - 11:10 Padmavathi Srinivasan (Boston University)
A canonical algebraic cycle associated to a curve in its Jacobian

11:20 - 12:10 David Zywina (Cornell University)
Galois representations of elliptic curves and modular curves

12:10 - 2:00 Lunch

Abstracts of talks

Group Theoretic Ax-Katz Theorems

Pete Clark

University of Georgia

Monday, June 3, 2024 @ 9:30 AM

The Chevalley-Warning Theorem is a result from 1935 asserting that the number of solutions to a low degree polynomial system over a finite field is divisible by the characteristic of the field. It is an important result – it includes a conjecture of Artin and Dickson from the 1920's – but it is not difficult to prove: the original proof is about three pages. In 1964, James Ax gave a completely elementary ten line proof. In the same paper, Ax showed that as the number and degrees of the polynomials are held fixed and the number of variables increases, not only is the size of the solution set divisible by p but by higher and higher powers of p. The best possible p-adic divisibility here was given in 1971 by Nicholas Katz. Katz's proof is at a much higher level: you need specialist knowledge in the right subfields of number theory to understand it. Simpler proofs were found later, but none fulfills the fantasy of generalizing Ax's ten line proof of Chevalley- Warning.
A 2021 work of Aichinger-Moosbauer develops a fully fledged calculus of finite differences for maps between commutative groups and uses it to give a purely group-theoretic generalization of Chevalley-Warning. Nicholas Triantafillou and I have used and extended this work: up to a few black boxes (where most of the content is indeed hidden) we give a ten line proof of a group-theoretic analogue of Ax-Katz that "qualitatively fulfills my fantasy."

Scheme theory over semirings and narrow class groups

James Borger

Australian National University

Monday, June 3, 2024 @ 11:10 AM

Usual scheme theory can be viewed as the syntactic theory of polynomial equations with integer coefficients. But none of its most fundamental ingredients, such as faithfully flat descent, require subtraction. So we can set up a scheme theory over semirings (rings but possibly without subtraction), thus bringing positivity in to the foundations of scheme theory. We might view positivity as integrality at the infinite place, the Boolean semiring as the residue field, and the positive reals as the completion.

In this talk, I'll discuss some recent developments in module theory over semirings. While the classical definitions of "vector bundle" are not all equivalent over semirings, all classical definitions of "line bundle" are equivalent, which allows us to define Picard groups and Picard stacks. The narrow class group of a number field can be recovered as the reflexive class group of the semiring of its totally nonnegative integers, bringing the narrow class group into scheme theory just as the ordinary class group was a long time ago.

This is based on joint work with Jaiung Jun.

Pseudo-null Modules in Iwasawa Theory

Ralph Greenberg

University of Washington

Monday, June 3, 2024 @ 2:00 PM

We will begin with a general discussion of so-called "pseudo-null" modules over the rings of interest in this talk (namely, formal power series rings over the p-adic integers). The specific modules that we plan to discuss were first studied in Iwasawa's classical theory about the behavior of ideal class groups in certain towers of number fields. We will discuss various questions and conjectures about these modules and recent attempts to say something nontrivial about their structure. In particular, we will describe recent joint work with F. Bleher, T. Chinburg, M. Kakde, R. Sharifi, and M. Taylor.

Distribution of Selmer groups in quadratic twist families

Carlo Pagano

Concordia University

Monday, June 3, 2024 @ 3:10 PM

I will overview some recent advancements in the study of the behavior of Selmer groups of a general Galois module, both in linear and non-linear quadratic twist families. This is based on joint work with S. Chan, P. Koymans and E. Sofos.

Large Gaps Between k-free Integers and Polynomial Analogs

Angel Kumchev

Towson University

Tuesday, June 4, 2024 @ 9:30 AM

The study of the shortest interval $(x, x+h]$ that contains squarefree integers for all sufficiently large x goes back to the 1940s. A period of active development in the 1980s and early 1990s culminated in a celebrated result of Filaseta and Trifonov that if c is a sufficiently large constant, one can take $h \le cx^{1/5} \log x$. Generalizations for integers free of kth powers, $k > 2$, were also obtained. However, since then progress has stalled.

In this talk, I will report on some related work completed by an REU group I supervised jointly with Nathan McNew in the summer of 2022. First, through a thorough examination and careful optimization of the techniques of Filaseta and Trifonov we were able to prove a fully explicit version of their gap result: The interval $(x, x + 11x^{1/5} \log x]$ contains a squarefree integer for all $x \ge 2$. We also proved versions with smaller explicit constants that hold when $x \ge x_0$, where x_0 is given explicitly, as well as explicit results on gaps between cubefree and general k-free integers by developing ideas of Trifonov from the mid 1990s.

Further, we considered analogs of the above gap problem for gaps between squarefree and k-free polynomials over a finite field $\mathbb{F}_q$. In this context, much stronger asymptotic bounds are known in the squarefree case from recent work of Cardon and Entin. We simplified and generalized their approach to gaps between k-free polynomials in $\mathbb{F}_q[x]$, for arbitrary $k \ge 2$. We also developed polynomial versions of the classical techniques used to study gaps between k-free integers in $\mathbb{Z}$, which we applied to obtain analogues in $\mathbb{F}_q[x]$ of some classical theorems on the distribution of k-free integers. The latter results complement our generalization of the Cardon–Entin theorem in the case when the degrees of the polynomials are of moderate size.

This is joint work with W. McCormick, N. McNew, A. Park, R. Scherr, and W. Ziehr.

Torsion finite problems

Jeff Achter

Colorado State University

Tuesday, June 4, 2024 @ 11:10 AM

Consider an abelian variety A over a number field K. The torsion subgroup of $A(K)$ is finite; a result of Ribet shows that this finiteness persists over the cyclotomic extension of K. Now consider a second abelian variety B/K, and the infinite extension K_B generated by the coordinates of its torsion points. Conditional on the Mumford-Tate conjecture (and up to a finite extension of K), I will give a criterion for the finitude of the torsion subgroup of $A(K_B)$. I'll also sketch out a conjectural motivic generalization of this story. Joint work with Lian Duan and Xiyuan Wang.

An equivariant Tamagawa number formula for abelian t–motives and applications

Cristian Popescu

University of California, San Diego

Tuesday, June 4, 2024 @ 2:00 PM

In fairly recent joint work with Ferrara, Green and Higgins we proved a Tamagawa number formula for special values of Galois equivariant Goss L–functions associated to Drinfeld modules and, more generally, to abelian t–motives. In this lecture, we will discuss this formula and give some applications, including (if time permits) a characteristic p Iwasawa theory for Taelman modules associated to abelian t–motives.

Hecke L-series for Sinha modules

Matt Papanikolas

Texas A&M University

Tuesday, June 4, 2024 @ 3:10 PM

There are several deep-seated analogies arising from the arithmetic of global function fields, especially the connections between elliptic curves and abelian varieties with their counterparts Drinfeld modules and Anderson t-modules. In 1997 Sinha introduced a class of Anderson t-modules with complex multiplication by rings of integers in Carlitz cyclotomic extensions. Using Anderson's theory of solitons and Coleman functions, he showed that the coordinates of periods of these t-modules could be expressed in terms of values of Thakur's geometric Gamma function at rational arguments. As part of these constructions, Anderson and Sinha also defined algebraic Hecke characters whose multiplication type is related to the CM-type of the t-module. We show that the Goss L-series for Sinha modules are products of these Hecke L-series, and we investigate their special values in terms of Gamma values.
Joint with E. Davis.

Hilbert's Tenth Problem for Some New Families of Number Fields

Florian Sprung

Arizona State University

Tuesday, June 4, 2024 @ 4:10 PM

Hilbert's Tenth Problem asks whether there is an algorithm that can decide whether a Diophantine (integral coefficients) equation has integral solutions. Matiyasevich in 1970 showed the answer in general was no. Denef and Lipshitz generalized Matiyasevich's theorem to 'Diophantine' equations with coefficients in rings of integers of some number fields, and conjectured that their result should hold for any number field. We present some new cases of their conjecture, building crucially on the connection with elliptic curves and concomitant work by Poonen, Shlapentokh, and Garcia-Fritz & Pasten.
This is joint work with D. Kundu and A. Lei.

Iwasawa-theoretic analogues of the Riemann–Hurwitz and Deuring–Shafarevich formulae

Bryden Cais

University of Arizona

Wednesday, June 5, 2024 @ 9:00 AM

A fundamental numerical invariant of a smooth projective curve X over a field k is its genus. In any branched Galois cover of such curves Y—> X over k with group G, the Riemann–Hurwitz formula expresses the genus of Y in terms of the genus of X, the order of G, and the ramification of the cover. When k has positive characteristic p, the Frobenius endomorphism of the cohomology group $H^1(O_X)$ provides a litany of refined numerical invariants of X. In this setting, when G is a p-group, the Deuring–Shafarevich formula is a beautiful analogue of the Riemann–Hurwitz formula for p-ranks, i.e. the dimension of the maximal F-stable subspace of $H^1(O_X)$ on which F is invertible. One would like an analogue of these celebrated formulae for the "rest" of $H^1(O_X)$, on which F acts nilpotently. Simple examples show that there can be no such exact formula in general. Nevertheless, in recent joint work with Booher, Kramer–Miller and Upton, we establish a new, Iwasawa-theoretic analogue of the Riemann–Hurwitz and Deuring–Shafarevich formulae in certain ramified Z_p-extensions of the rational function field over a finite field of characteristic p. This talk will give a survey of this work and some related ideas and questions.

The p-adic valuation of local resolvents and anticyclotomic Hecke L-values of imaginary quadratic fields at inert prime

Shinichi Kobayashi

Kyushu University

Wednesday, June 5, 2024 @ 10:20 AM

We explain an asymptotic formula for the p-adic valuation of Hecke L-values of an imaginary quadratic field at an inert prime p along the anticyclotomic Z_p-tower. The key is the determination of the p-adic valuation of local resolvents in Z_p-extensions.

Non-zero Central Values of Dirichlet Twists of Elliptic L-Functions

Hershy Kisilevsky

Concordia University

Wednesday, June 5, 2024 @ 11:20 AM

We consider heuristic predictions for "small" non-zero algebraic central values of twists of the L-function of an elliptic curve E/Q by Dirichlet characters. We provide computational evidence for these predictions and some consequences for an analogue of the Brauer-Siegel Theorem in this context.

Dominant Galois Groups for Large Degree Number Fields

Jiuya Wang

University of Georgia

Thursday, June 6, 2024 @ 9:30 AM

It is classical that Hilbert irreducibility theorem implies that 100% of degree n polynomials with bounded coefficients are irreducible and have Galois group S_n. One can instead ask what the dominant Galois group is when we order the number fields by their discriminants. For small degrees (less or equal to 5), the full symmetric group S_n is still the dominant Galois group. We will investigate this question for large n. This is a joint work with Daniel Keliher.

Iwasawa theory of isogeny graphs with level structures

Antonio Lei

University of Ottawa

Thursday, June 6, 2024 @ 11:10 AM

Let p be a prime number. In classical Iwasawa theory, we are interested in the study of arithmetic objects arising from p-adic Lie extensions of number fields. Recently, several concepts originating from classical Iwasawa theory have found extension to towers of graph coverings. This talk will begin with a review of these recent advances. I will explain how isogeny graphs, enhanced with level structures, provide explicit examples that fit in the framework of Iwasawa theory of graph coverings. The talk will conclude with an overview of the "volcanic" structure of these graphs. This is joint work with Katharina Mueller.

Iwasawa Theory and Arithmetic Statistics

Debanjana Kundu

University of Texas - Rio Grande Valley

Thursday, June 6, 2024 @ 2:00 PM

In this talk, I will discuss questions which lie at the intersection of Iwasawa theory and arithmetic statistics. I will talk about some recent results in this direction and some potentially interesting questions.

On the Algebraic Torus associated to Totally Real Fields and the Shintani Generating Class

Kenichi Bannai

Keio University

Thursday, June 6, 2024 @ 3:20 PM

Classical Dirichlet L-function associated to a Dirichlet character may be expressed as a linear sum of Lerch Zeta Functions, and there exists a rational function on the multiplicative group which is a universal generating function for special values of the Lerch Zeta Functions at various integer points and roots of unity. Generalizing this result to the case of totally real fields, we re-interpret the classical result of Takuro Shintani to construct a certain universal equivariant class which we call the Shintani Generating Class on an algebraic torus associated to a totally real field which universally generates the special values of the Lerch Zeta Function at various integer points and torsion points of the algebraic torus. We then give some observation relating the algebraic torus and the arithmetic of the totally real field. This is a joint work with H. Bekki, K. Hagihara, T. Oshita, K. Yamada, and S. Yamamoto.

Irrationality and linear independence of p-adic zeta values

Johannes Sprang

University of Duisburg-Essen

Friday, June 7, 2024 @ 9:00 AM

The celebrated theorem of Ball-Rivoal shows that there are infinitely many irrational odd zeta values. More precisely, it provides asymptotically lower bounds on the dimension of the Q-vector space spanned by these numbers.
Although we are still far from fully understanding the structure of odd zeta values, the corresponding question for p-adic zeta values is even more difficult. For example, the question of the non-vanishing of these numbers is still open today, although it has interesting consequences e.g. for the non-vanishing of p-adic regulators. In this talk we would like to report recent progress on the irrationality and linear independence of p-adic zeta values.
Parts of this talk are based on joint work with Li Lai.

A canonical algebraic cycle associated to a curve in its Jacobian

Padmavathi Srinivasan

Boston University

Friday, June 7, 2024 @ 10:20 AM

The Ceresa cycle is a canonical homologically trivial algebraic cycle associated to a curve in its Jacobian. In his 1983 thesis, Ceresa showed that this cycle is algebraically nontrivial for the generic curve over genus at least 3. Strategies for proving Fermat curves have infinite order Ceresa cycles due to B. Harris, Bloch, Bertolini-Darmon-Prasanna, Eskandari-Murty use a variety of ideas ranging from computation of explicit iterated period integrals, special values of p-adic L functions and points of infinite order on the Jacobian of Fermat curves. We will survey many recent results around the Ceresa cycle, and present ongoing work with Jordan Ellenberg, Adam Logan and Akshay Venkatesh where we produce many new explicit examples of curves over number fields with infinite order Ceresa cycles.

Galois representations of elliptic curves and modular curves

David Zywina

Cornell University

Friday, June 7, 2024 @ 11:20 AM

For an elliptic curve E defined over the rational numbers, the Galois action on its torsion points encodes almost all of its arithmetic. A famous theorem of Serre says if E is non-CM, then the image of the corresponding Galois representation is as large as possible up to commensurability. We shall describe recent methods to explicitly compute the images of these representations via modular curves.

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

Jeff Achter, Colorado State University
Josue Avila Artavia, University of Maryland
Kenichi Bannai, Keio University
Dori Bejleri, University of Maryland
James Borger, Australian National University
Patrick Brosnan, University of Maryland
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Harry Tamvakis, University of Maryland
Jiuya Wang, University of Georgia
Larry Washington, University of Maryland
Melanka Wedige, University of Maryland
David Zywina, Cornell University

BRIN MATHEMATICS RESEARCH CENTER

Academic Year 2023–24

The Brin Mathematics Research Center was established in 2022 through a generous gift from the Brin family, as a research center that sponsors activities in all areas of mathematics and statistics. The Brin MRC is managed by the Department of Mathematics at the University of Maryland, College Park.

Fall 2023

- **Polylogarithms, Cluster Algebras, and Scattering Amplitudes**
  (9/11/2023–9/15/2023)
- **Low Complexity Dynamical Systems**
  (10/2/2023–10/6/2023)
- **Statistical Inference on Networks and High-Dimensional Data**
  (10/18/2023–10/20/2023)
- **Mathematics of Malaria Transmission Dynamics**
  (11/13/2023–11/17/2023)

Spring 2024

- **Applied Stochastic Processes for Encounter Problems**
  (2/5/2024–2/9/2024)
- **Scientific Machine Learning: Theory and Algorithms**
  (2/21/2024–2/23/2024)
- **Mathematical Models of Electronic Transport and Phases in Low-Dimensional Materials**
  (3/11/2024–3/15/2024)
- **Recent Advances in Time Series Analysis**
  (3/21/2024–3/22/2024)
- **Advances in Higgs Bundles**
  (4/15/2024–4/19/2024)
- **Several Complex Variables, Complex Geometry and related PDEs**
  (5/13/2024–5/17/2024)

Summer 2024

- **Vistas In Number Theory**
  (6/3/2024–6/7/2024)
- **Summer School:** Tracer Mixing in Fluids Across Plaetary Scales
  (7/8/2024–7/19/2024)
- **Summer School:** PDE and Randomness
  (8/5/2024/–8/9/2024)