BRIN MATHEMATICS RESEARCH CENTER

WEYL LAWS
ACROSS MATHEMATICS

MAY 5 -9, 2025

ORGANIZERS

Dan Cristofaro-Gardiner, University of Maryland
Rohil Prasad, UC Berkeley
Xin Zhou, Cornell University

ABOUT THE WORKSHOP

There has been much recent progress on the Weyl-type asymptotic results for linear and non-linear spectra, with striking applications across several fields. The goal of this workshop is to bring together experts in these fields to interact outside their regular mathematics circles and make new connections.

PARTICIPANTS

Lucas Ambrozio, IMPA
Adrian Chu, MSRI/SLMath
Semyon Dyatlov, MIT
Oliver Edtmair, ETH Zürich
Richard Hind, University of Notre Dame
Helmut Hofer, Institute for Advanced Study
Xiaoqi Huang, Louisiana State University
Michael Hutchings, UC Berkeley
Mikhail Karpukhin, University College, London
Simon Larson, University of Gothenburg
Nicki Magill, UC Berkeley
Christos Mantoulidis, Rice University
Liam Mazurowski, Cornell University
Andre Neves, University of Chicago
Christopher Sogge, Johns Hopkins University
Bruno Staffa, University of Toronto
Zhihan Wang, Cornell University
Morgan Weiler, UC Riverside
Xuwen Zhu, Northeastern University

CSIC Building, 4th Floor
8169 Paint Branch Drive
University of Maryland
College Park, MD 20742

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DEPARTMENT OF MATHEMATICS

Contents

Schedule at a Glance

	Monday	Tuesday	Wednesday	Thursday	Friday
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10:00	Hutchings	Dyatlov	Neves	Sogge	Hind
11:00	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
12:00	Mantoulidis	Edtmair	Ambrozio	Zhu	Zhang
13:00	Lunch	Lunch	Li	Lunch	
14:00	Shelukhin	Huang		Larson	
15:00	Coffee Break	Coffee Break		Coffee Break	
16:00	Mazurowski	Chu		Staffa	
17:00	Marx-Kuo	Problem Session		Magill	
18:00					

Workshop Overview

There has been much recent progress on the Weyl-type asymptotic results for linear and nonlinear spectra, with striking applications across several fields. The goal of this workshop is to bring together experts in these fields to interact outside their regular mathematics circles and make new connections.

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Workshop Schedule

Monday, May 5, 2025

8:50 - 9:20 Breakfast

9:20 - 9:30 Doron Levy (University of Maryland/Director, Brin MRC)
Opening

9:40 - 10:40 Michael Hutchings (University of California, Berkeley)
Reeb Orbits Frequently Intersecting a Symplectic Surface

10:40 - 11:10 Coffee Break

11:10 - 12:10 Christos Mantoulidis (Rice University)
The p-widths of S^2

12:10 - 1:30 Lunch

1:30 - 2:30 Egor Shelukhin (University of Montreal)
Coarse Nodal Count and Topological Persistence

2:30 - 3:00 Coffee Break

3:00 - 4:00 Liam Mazurowski (Cornell University)
Half Volume Spectrum and CMC Hypersurfaces

4:30 - 5:30 Jared Marx-Kuo (Rice University)
Infinitely Many Surfaces with Prescribed Mean Curvature in the Presence of a Strictly Stable Minimal Surface

Tuesday, May 6, 2025

9:00 - 9:30	Breakfast
9:30 - 10:30	Semyon Dyatlov (Massachusetts Institute of Technology) *Control of Eigenfunctions on Negatively Curved Manifolds*
10:30 - 11:00	Coffee Break
11:00 - 12:00	Oliver Edtmair (ETH Zurich) *The Subleading Asymptotics of Symplectic Weyl Laws*
12:00 - 1:30	Lunch
1:30 - 2:30	Xiaoqi Huang (Louisiana State University) *Strichartz Estimates for the Schrodinger Equation on the Sphere*
2:30 - 3:00	Coffee Break
3:00 - 4:00	Adrian Chu (MSRI/SLMath) *On Min-Max Widths and Minimal Surfaces in 3-Spheres*
4:00 - 5:30	Problem session
6:30 - 8:30	Conference Dinner

Wednesday, May 7, 2025

8:50 - 9:20 Breakfast

9:20 - 10:20 Andre Neves (The University of Chicago)
Weyl Law and Minimal Surfaces

10:20 - 10:50 Coffee Break

10:50 - 11:50 Lucas Ambrozio (Instituto Nacional de Matemática Pura e Aplicada)
Constructions of Zoll-Like Metrics in Minimal Surface Theory

12:00 - 1:00 Xingzhe Li (Cornell University)
Generic Scarring for Minimal Hypersurfaces in Manifolds Thick at Infinity with a Thin Foliation at Infinity

Thursday, May 8, 2025

9:00 - 9:30 Breakfast

9:30 - 10:30 Christopher Sogge (Johns Hopkins University)
Curvature and Harmonic Analysis on Compact Manifolds

10:30 - 11:00 Coffee Break

11:00 - 12:00 Xuwen Zhu (Northeastern University)
Spectral Gaps for Large Genus Hyperbolic Surfaces

12:00 - 1:30 Lunch

1:30 - 2:30 Simon Larson (University of Gothenburg)
Refined Weyl Asymptotics in Rough Geometries

2:30 - 3:00 Coffee Break

3:00 - 4:00 Bruno Staffa (University of Toronto)
Weyl Law for 1*-Cycles and Applications*

4:30 - 5:30 Nicki Magill (University of California, Berkeley)
Generalized Convex Toric Domains and Symplectic Embedding Problems

Friday, May 9, 2025

9:00 - 9:30 Breakfast

9:30 - 10:30 Richard Hind (University of Notre Dame)
A Weyl Law and the Failure of Symplectic Packing Stability

10:30 - 11:00 Coffee Break

11:00 - 12:00 Boyu Zhang (University of Maryland)
The Smooth Closing Lemma for Area-Preserving Diffeomorphisms of Surfaces

Abstracts of talks

Reeb Orbits Frequently Intersecting a Symplectic Surface

Michael Hutchings

University of California, Berkeley

Monday, May 5, 2025 @ 9:40 AM

We present a recent application of Weyl laws for ECH-type spectral invariants of contact three-manifolds and symplectic four-manifolds. Consider a symplectic surface in a three-dimensional contact manifold with boundary on elliptic Reeb orbits. We show that assuming a certain condition on the rotation number of the boundary Reeb orbits, there must exist Reeb orbits which intersect the interior of the surface with a certain lower bound on the frequency. As a corollary, we obtain a generalization of some recent results relating the mean action to the Calabi invariant for area-preserving surface diffeomorphisms.

The p-widths of S^2

Christos Mantoulidis

Rice University

Monday, May 5, 2025 @ 11:10 AM

The p-widths of a closed Riemannian manifold are a nonlinear geometric analog of the spectrum of its Laplace–Beltrami operator. Their study was initiated by Gromov, Guth, and broadened by Marques and Neves. By a theorem of Liokumovich–Marques–Neves, the p-widths obey a Weyl law just like eigenvalues do. However, even though eigenvalues are explicitly computable for many manifolds, the p-widths appear particularly elusive to compute. I'll discuss work with Otis Chodosh, in which we found all p-widths on the round 2-sphere and thus computed the previously unknown Liokumovich–Marques–Neves Weyl law constant in dimension 2.

Coarse Nodal Count and Topological Persistence

Egor Shelukhin

University of Montreal

Monday, May 5, 2025 @ 1:30 PM

Courant's classical nodal domain theorem, together with the Weyl law for Laplace eigenvalues, bounds the number of connected components of the non-zero set of a Laplace eigenfunction in terms of the n/2-th power of its eigenvalue. Natural extensions of this statement, such as the Courant-Herrmann conjecture for linear combinations of eigenfunctions, and counts of higher Betti numbers, turn out to fail. However, we prove using the theory of persistence modules and polynomial approximation that they still hold if one ignores small oscillations. This talk is based on a joint work with L. Buhovsky, J. Payette, I. Polterovich, L. Polterovich, and V. Stojisavljevic.

Half Volume Spectrum and CMC Hypersurfaces

Liam Mazurowski

Cornell University

Monday, May 5, 2025 @ 3:00 PM

Let M be a closed, three dimensional Riemannian manifold. A constant mean curvature surface in M is a critical point of the area functional subject to a volume constraint. The solution to the isoperimetric problem implies that, for each v between 0 and the volume of M, there is a constant mean curvature surface in M enclosing volume v. In this talk, we show that in the special case where v is half the volume of M and the metric on M is generic, there actually exist infinitely many distinct constant mean curvature surfaces cutting M into two pieces of equal volume. The construction depends in a crucial way on a Weyl law for a modified volume spectrum. Everything in this talk is joint work with Xin Zhou.

Infinitely Many Surfaces with Prescribed Mean Curvature in the Presence of a Strictly Stable Minimal Surface

Jared Marx-Kuo

Rice University

Monday, May 5, 2025 @ 4:30 PM

We construct infinitely many distinct hypersurfaces with prescribed mean curvature (PMC) for a large class of prescribing functions on closed manifolds, (M^{n+1}, g), containing a strictly stable minimal hypersurface. Such a hypersurface exists when $H_n(M^{n+1}) \neq 0$ or if (M^{n+1}, g) does not satisfy the Frankel property. Our construction synthesizes ideas from Song's construction of infinitely many minimal surfaces in the non-generic setting, Dey's construction of multiple constant mean curvature surfaces, and Sun–Wang–Zhou's min-max construction of free boundary PMCs. A key tool is the linear Weyl Law for Gromov's p-widths for a manifold with cylindrical ends.

Control of Eigenfunctions on Negatively Curved Manifolds

Semyon Dyatlov

Massachusetts Institute of Technology

Tuesday, May 6, 2025 @ 9:30 AM

Semiclassical measures are a standard object studied in quantum chaos, capturing macroscopic behavior of sequences of eigenfunctions in the high energy limit. They have a long history of study going back to the Quantum Ergodicity theorem and the Quantum Unique Ergodicity conjecture. I will speak about the work with Jin and Nonnenmacher, proving that on a negatively curved surface, every semiclassical measure has full support. I will also discuss applications of this work to control for the Schrodinger equation and decay for the damped wave equation.
Our theorem was restricted to dimension 2 because the key new ingredient, the fractal uncertainty principle (proved by Bourgain and myself), was only known for subsets of the real line. I will talk about more recent joint work with Athreya and Miller in the setting of complex hyperbolic quotients and the work in progress by Kim and Miller in the setting of real hyperbolic quotients of any dimension. In these works there are potential obstructions to the full support property which can be classified by Ratner theory and geometrically described in terms of certain totally geodesic submanifolds. Time permitting, I will also mention a recent counterexample to Quantum Unique Ergodicity for higher-dimensional quantum cat maps, due to Kim and building on the previous counterexample of Faure-Nonnenmacher-De Bievre.

The Subleading Asymptotics of Symplectic Weyl Laws

Oliver Edtmair

ETH Zurich

Tuesday, May 6, 2025 @ 11:00 AM

It is an interesting problem to obtain refined quantitative estimates on the remainder terms of a Weyl law. In the case of the classical Weyl law for the Laplace spectrum, numerous authors have made contributions to this question over the course of the last century. In my talk, I will explain how to obtain sharp asymptotic estimates on the remainder terms of various Weyl laws in symplectic geometry. I will also explain how this is related to the phenomenon of symplectic ball packing stability and the recently resolved simplicity conjecture.

Strichartz Estimates for the Schrodinger Equation on the Sphere

Xiaoqi Huang

Louisiana State University

Tuesday, May 6, 2025 @ 1:30 PM

We will discuss optimal space-time estimates in $L^q_{t,x}$ spaces for solutions to the Schrodinger equation on the standard round sphere, which is related to the results of Burq, Gerard and Tzvetkov (2004). The proof is based on the arithmetic properties of the spectrum of the Laplacian on the sphere, as well as local bilinear oscillatory integral estimates in harmonic analysis, which allow us to relate the problem to Strichartz estimate on one-dimensional tori. This is based on joint work with Christopher Sogge.

On Min-Max Widths and Minimal Surfaces in 3-Spheres

Adrian Chu

MSRI/SLMath

Tuesday, May 6, 2025 @ 3:00 PM

We will survey some results regarding min-max widths of the unit 3-sphere. We will also present enumerative problems regarding the construction of minimal surfaces of specific genus. In particular, we will introduce our recent proof of the existence of 5 minimal tori in every 3-sphere of positive Ricci curvature.

Weyl Law and Minimal Surfaces

Andre Neves

The University of Chicago

Wednesday, May 7, 2025 @ 9:20 AM

I will talk about the impact the Weyl Law had on the existence theory for minimal surfaces.

Constructions of Zoll-Like Metrics in Minimal Surface Theory

Lucas Ambrozio

Instituto Nacional de Matemática Pura e Aplicada

Wednesday, May 7, 2025 @ 10:50 AM

Zoll metrics are Riemannian metrics all of whose geodesics are closed, simple and have the same length. Analogues of these objects in the theory of minimal surfaces are emerging as interesting objects, which are, for instance, detected by the behavior of the area spectrum of manifolds, that is, by the behavior of certain min-max critical values of the area functional on the space of hypersurfaces. Our talk will be dedicated to discuss constructions of these objects and of their basic properties. This is part of a collaboration with D. Guajardo, and F. Coda Marques and A. Neves.

Generic Scarring for Minimal Hypersurfaces in Manifolds Thick at Infinity with a Thin Foliation at Infinity

Xingzhe Li

Cornell University

Wednesday, May 7, 2025 @ 12:00 PM

Generically, variationally defined geometric objects often either equidistribute, or concentrate ("scar") along a proper subset. In this talk, we present a generic scarring phenomenon for minimal hypersurfaces in smooth manifolds. As part of the discussion, we will introduce the Weyl law for the single-cylindrical volume spectrum, and outline a perturbation argument applied to it.

Curvature and Harmonic Analysis on Compact Manifolds

Christopher Sogge

Johns Hopkins University

Thursday, May 8, 2025 @ 9:30 AM

We shall explore the role that curvature plays in harmonic analysis on compact manifolds. We shall focus on estimates that measure the concentration of eigenfunctions. Using them, we are able to affirm the classical Bohr correspondence principle and obtain a new classification of compact space forms in terms of the growth rates of various norms of (approximate) eigenfunctions.
This is joint work with Xiaoqi Huang following earlier work with Matthew Blair.

Spectral Gaps for Large Genus Hyperbolic Surfaces

Xuwen Zhu

Northeastern University

Thursday, May 8, 2025 @ 11:00 AM

The study of "small" eigenvalues ($<1/4$) of the Laplacian on hyperbolic surfaces has a long history and has recently seen many developments. In this talk, I will discuss the higher spectral gaps, where we study the differences of consecutive eigenvalues up to λ_{2g-2} for genus g hyperbolic surfaces. I will focus on the joint work with Yunhui Wu and Haohao Zhang where we showed that the supremum of such spectral gaps over the moduli space has infimum limit at least $1/4$ as genus goes to infinity.

Refined Weyl Asymptotics in Rough Geometries

Simon Larson

University of Gothenburg

Thursday, May 8, 2025 @ 1:30 PM

The classical Weyl's law describes the leading-order asymptotic behaviour of the eigenvalues of Laplace operators in a bounded open set. Weyl conjectured that the asymptotic law could be refined and that a subleading term should exist. If the underlying geometry is sufficiently smooth, this conjecture was famously settled by Victor Ivrii in 1980 under an assumption on the billiard dynamics in the underlying set. In this talk we will explore some recent results in this direction with a view towards geometries of limited regularity. In particular, I will discuss two-term Weyl asymptotics for certain averages of the eigenvalues of the Dirichlet, Neumann, or Robin Laplace operators under the assumption that the boundary of the underlying set is Lipschitz regular. Based on joint work with Rupert Frank (LMU Munich).

Weyl Law for 1-Cycles and Applications

Bruno Staffa

University of Toronto

Thursday, May 8, 2025 @ 3:00 PM

Let (M^n, g) be an n-dimensional Riemannian manifold and $1 \leq k \leq n-1$. In the 1980's, Gromov studied the asymptotic behavior of the volumes $\omega_p^k(M^n, g)$ of certain (possibly singular) k-dimensional minimal submanifolds N_p^k of (M^n, g) arising from a Morse Theory of the volume functional (the Almgren-Pitts Min-Max Theory). He conjectured that these volumes $\omega_p^k(M^n, g)$ behave when $p \to \infty$ similarly to the eigenvalues λ_p of the Laplacian in (M^n, g), obeying the so-called Weyl Law for the Volume Spectrum. I will discuss the current progress on that conjecture, with emphasis on the 1-cycles case and its applications to generic density and equidistribution of closed geodesics and stationary geodesic nets.

Generalized Convex Toric Domains and Symplectic Embedding Problems

Nicki Magill

University of California, Berkeley

Thursday, May 8, 2025 @ 4:30 PM

A convex toric domain X_Ω is a 4-dimensional subset of $\mathbb{R}^4$, defined as the preimage of a bounded convex region Ω in the positive quadrant of $\mathbb{R}^2$ under the moment map. We consider how geometric features of Ω such as the curviness of its boundary and its affine perimeter impact symplectic packing problems. Some of our results come from considering the asymptotics of the ECH capacities

$$\{c_k(X_\Omega)\}_{k\in Z_{\geq 0}}$$

. These capacities are known to obey a Weyl law and thus detect the volume of X_Ω. We show that their subleading asymptotics detect the affine perimeter of Ω. We'll discuss how these asymptotic results lead to new applications in symplectic embedding problems. This is based on upcoming work with Dan Cristofaro-Gardiner and Dusa McDuff.

A Weyl Law and the Failure of Symplectic Packing Stability

Richard Hind

University of Notre Dame

Friday, May 9, 2025 @ 9:30 AM

We describe some embedding and packing problems in symplectic topology, and obstructions coming from a Weyl law which, given a relatively compact domain in a symplectic 4 manifold, relates the subleading asymptotics of the Embedded Contact Homology capacities and the inner Minkowski dimension. For certain toric examples we obtain sharp bounds on the possible Minkowski dimension of symplectomorphic domains. This describes joint work with Dan Cristofaro-Gardiner.

The Smooth Closing Lemma for Area-Preserving Diffeomorphisms of Surfaces

Boyu Zhang

University of Maryland

Friday, May 9, 2025 @ 11:00 AM

In this talk, I will introduce a proof of the smooth closing lemma for area-preserving diffeomorphisms on surfaces. The proof is based on a Weyl formula for the spectral invariants of Periodic Floer Homology and a non-vanishing result of twisted Seiberg-Witten Floer Homology. This is joint work with Dan Cristofaro-Gardiner and Rohil Prasad.

The Brin Mathematics Research Center

The Brin Mathematics Research Center is a research center that sponsors activity in all areas of pure and applied mathematics and statistics. The Brin MRC was funded in 2022 through a generous gift from the Brin Family. The Brin MRC is part of the Department of Mathematics at the University of Maryland, College Park.

Activities sponsored by the Brin MRC include long programs, conferences and workshops, special lecture series, and summer schools. The Brin MRC provides ample opportunities for short-term and long-term visitors that are interested in interacting with the faculty at the University of Maryland and in experiencing the metropolitan Washington DC area.

The mission of the Brin MRC is to promote excellence in mathematical sciences. The Brin MRC is home to educational and research activities in all areas of mathematics. The Brin MRC provides opportunities to the global mathematical community to interact with researchers at the University of Maryland. The center allows the University of Maryland to expand and showcase its mathematics and statistics research excellence nationally and internationally.

List of Participants

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Zhihan Wang, Cornell University
Morgan Weiler, University of California, Riverside
Boyu Zhang, University of Maryland
Xin Zhou, Cornell University
Xuwen Zhu, Northeastern University